14

A Conceptual Model Discussed by Galileo and Used Intuitively by Physics Students

John Clement
University of Massachusetts

Galileo's works are notable for their use of two interesting techniques: the use of qualitative thought experiments for presenting basic concepts and theories, and the inclusion and consideration of theories Galileo considered to be wrong, as voiced by Simplicio in the *Dialogues*(Galileo, 1960). One wonders why Galileo did not do as modern physics texts do and simply present mathematical results. Presumably, Galileo recognized that it was going to be difficult to present his views convincingly to his colleagues, since they subscribed to a more Aristotelian approach that essentially amounted to a very different "world view." It is plausible that he felt it necessary to discuss and counter the preconceptions of his colleagues explicitly, and to use qualitative arguments in presenting his own theory, because he sensed the strong resistance of their world view to change.

We have collected data that indicate many students have a significantly different view of certain aspects of mechanics than the now standard Newtonian view. Furthermore, this view, like those of Galileo's colleagues, appears to be fairly resistant to change and persists even after taking college physics. Specifically, many students have an alternative mental model for the relationship between force and motion, probably based on their own intuitions about how to move objects around in the world. This model conflicts with the qualitative model of the physicist underlying the equation $F = ma$. When a constant force is applied to an object, the physicist thinks of this as producing a constant acceleration in the same direction as the force. Such a model is a "conceptual primitive" in the sense that it is a basic prerequisite for learning many higher-order principles in physics.

The student's intuitive model is usually structured differently than the physicist's. In the real world, where friction is present, one must push on an object to keep it moving. Because friction is often not recognized by the beginner as a force, the student may believe that continuing motion, even at a constant speed, implies the presence of a continuing force in the same direction, as a necessary cause of the motion. We call this the "motion implies a force" misconception. Empirical evidence is presented indicating that many beginners apply this view to various simple mechanics problems. In fact, the misconception shows up in a wider diversity of problem situations than one would expect, and appears to still be present in many students after they have completed a course in mechanics. It therefore appears to be a major stumbling block in the physics curriculum. Related misconceptions have been studied by Driver (1973), Viennot (1979), Lawson, Trowbridge, and McDermott (1979), and DiSessa (1979). In this study it is shown that preconceptions can be studied using problems of minimum complexity which help to isolate the source of the errors. It was discovered that the motion implies a force preconception and is remarkably similar to a conception discussed by Galileo. This is illustrated by a comparison of his writings to transcripts of student interviews.

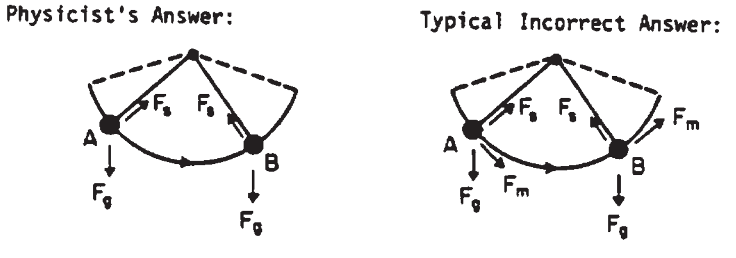
THE "MOTION IMPLIES A FORCE" PRECONCEPTION

The error pattern described in the following example was observed in a large number of course laboratory write-ups from students taking introductory mechanics after they had worked with pendulums in the lab. A typical incorrect solution to the Pendulum Problem is shown in Fig. 14.1.

Example 1: Pendulum Problem

Question: (1) A pendulum is swinging from left to right as shown below. Draw arrows showing the direction of each force acting on the pendulum bob at point A. Do *not* show the total net force and do not include frictional forces. *Label each arrow* with a name that says what kind of force it is.

(2)In a similar way, draw and label arrows showing the direction of each force acting on the pendulum bob when it reaches point B.


Physicist's Answer:

Typical Incorrect Answer:

Fig. 14.1. Correct and incorrect answers to pendulum problem.

Typical Incorrect Explanation. F_m is the force that makes the pendulum swing upward. If F_m weren't there, the pendulum could never move up to the top of its swing.

Here, F_m is seen as one of the forces acting on the bob and is often described as the force that "makes the pendulum go up on the other side." We also noticed that students drawing force diagrams for an object sliding down a track, or for an object in orbit, would often include a force in the direction of motion. These classroom observations led us to suspect that many students were applying the idea that continuing motion implies the presence of a continuing force in the same direction as the motion. This type of belief shows up in pre-Newtonian theories of motion such as an impetus force injected into an arrow and travelling with it, or the Aristotelian explanation of the horizontal motion of an arrow after release from the bow via forward forces from air currents. What is surprising is the pervasiveness of the belief and the wide diversity of situations in which it shows up, once one begins to listen to students' common-sense theories. [For a summary of different impetus theories, see Dyksterhuis (1961) or Franklin (1978).]

In an effort to further isolate the source of this type of error, we designed the problem shown in Fig. 14.2, and predicted that it might trigger the "motion implies a force" misconception in spite of its extreme simplicity.

Example 2: Coin Problem

A coin is tossed from point A straight up into the air and caught at point E. On the dot to the left of the drawing draw one or more arrows showing the direction of each force acting on the coin when it is at point B. (Draw longer arrows for larger forces.)

Typical Incorrect Answer. While the coin is on the way up, the "force from your hand", F_h, pushes up on the coin. On the way up it must be greater than F_g, otherwise the coin would be moving down.

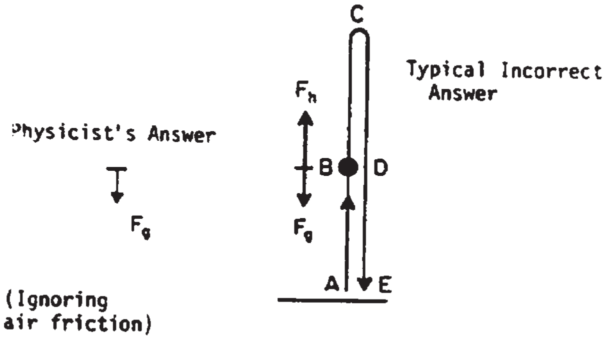


Fig. 14.2. Correct and incorrect answers to coin problem.

TABLE 14.1
Performance on Coin and Rocket Problems. Scores From Three Separate Groups of Engineering Majors Before and After Taking Mechanics

		Percentage Correct Before Course	*Percentage Correct After Course*	*Percentage Correct (For engineers with 2 semesters of Physics at a Second Institution)*
COIN PROBLEM		12% (N = 34)	28% (N = 43)	30% (N = 37)
ROCKET PROBLEM	Part 1	11% (N = 150)	23% (N = 43)	35% (N = 37)
	Part 2	38% (N = 150)	72% (N = 43)	65% (N = 37)

On a diagnostic test we gave the coin problem to a representative group of engineering students early in their first semester before they had taken physics. We also gave it to two groups of engineering students after they had taken mechanics.[1] The results are shown in Table 14.1. Eighty-eight percent of the prephysics students gave an incorrect answer. Virtually all (90%) of the errors in this case involved showing an arrow labeled as a force pointing upwards at position B. Eleven students were interviewed while solving this problem aloud: five of these had taken a physics course in mechanics for scientists and engineers. Three students solved this problem correctly, whereas seven students drew an upward arrow at point B, referring to it as the "force of the throw," the "upward original force," the "applied force," the "force that I'm giving it," "velocity is pulling upwards, so you have a net force in this direction (points upwards)," the force up from velocity," and "the force of throwing the coin up." Another student gave a questionable response, referring to "a momentum force...acting up", which doesn't belong in "a formal free body diagram" but "is definitely a force." The latter three responses were from students who had taken the mechanics course. All of these students were engineering majors. Again, we see that it is difficult for the student to think about an object continuing to move in one direction with the total net force acting in a different direction. These findings supported our hypothesis that the "motion implies a force" preconception was involved in the students' responses to these problems.

Another example is provided by the Rocket problem shown in Fig. 14.3.

---

[1] This sample was chosen in part because engineering students comprise the largest clientele of physics departments at many universities.

Example 3: Rocket Problem

(1) A rocket is moving along sideways in deep space, with its engine off, from point A to point B. It is not near any planets or other outside forces. Its engine is fired at point B and left on for two seconds while the rocket travels from point B to some point C. Draw in the shape of the path from B to C. (Show your best guess for this problem even if you are unsure of the answer).

(2)Show the path from point C after the engine is turned off on the same drawing.

Typical Incorrect Answer. The force of the rocket engine combines with whatever was making it go from A to B to produce path BC. After C, whatever made it go from A to B will take over and make it go sideways again, causing the rocket to return to its original direction of motion.

Results from written testing on this problem with a representative group of 150 prephysics engineering students are shown in Table 14.1. Eighty-nine percent drew an incorrect path for part 1 of the Rocket problem whereas 62% missed part 2. A summary of the responses to the Rocket problem is given in Table 14.2.

The curved path from B to C is a detailed aspect of the motion that the uninitiated student will rarely reproduce. A more surprising and significant difficulty than this, however, is the tendency in many students to actually draw the rocket's motion returning to a horizontal direction after the engine is shut off at point C. The student's prediction that the rocket will return to a horizontal path is usually accompanied by a reference to some influence acting on the rocket from A to B which "takes over" again after C. This behavior can be explained by assuming that, for the student, the presence of constant motion from A to B implies the presence of a continuing force in the same direction, even though the problem states that no outside forces are present. Note also that students usually show the direction of motion changing instantaneously in a noncontinuous manner, apparently to correspond to instantaneous changes in the direction of applied force.

Taped interviews were conducted with 18 of the above students. Five of the seven students who had responses of type 3 or 4 in Table 14.2 made a specific reference to the idea that "whatever was making it go to the right before will take

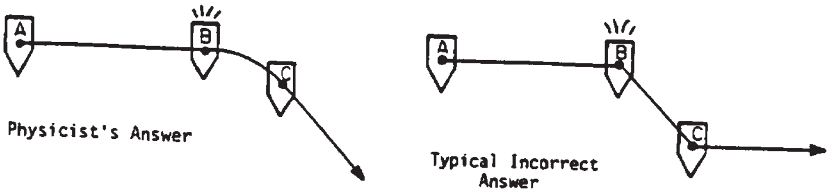


Fig. 14.3. Correct and incorrect answers to rocket problem.

tured very differently. When students with these alternative knowledge structures produce incorrect answers in the classroom, the instructor may in many cases assume that the cause is "low intelligence" or poorly developed reasoning skills, when in fact the cause is the stability of the student's alternative knowledge structures. It is important for teachers to become sensitive to such distinctions because the indicated teaching strategies are quite different in each case. Avoiding this confusion might have an impact on the way teachers view students and, in turn, on the way students view themselves.

ACKNOWLEDGMENT

Research reported in this paper was supported by NSF Award No. SED 78-22043 in the Joint National Institute of Education/National Science Foundation Program of Research on Cognitive Processes and the Structure of Knowledge in Science and Mathematics. Some of the material presented in this paper appears in "Students' preconceptions in introductory mechanics." *American Journal of Physics*, 1982, *50(1)*, 60–71.

REFERENCES

Arons, A. *The various language*. New York: Oxford University Press 1977.

Barowy, W., & Lochhead, J. Abstract reasoning in rotational physics. *American Association of Physics Teachers Announcer*, 1980, *10(2)*, 74.

Clement, J. Mapping a student's causal conceptions from a problem solving protocol. In J. Lochhead & J. Clement (Eds.), *Cognitive process instruction*. Philadelphia, Pennsylvania: Franklin Institute Press, 1979.(a)

Clement, J. *The role of analogy in scientific thinking: Examples from a problem solving interview*. Technical Report, Cognitive Development Project, Department of Physics and Astronomy, University of Massachusetts at Amherst, 1979.(b)

Clement, J. *Analogy generation in scientific problem solving*. Third Annual Meeting of the Cognitive Science Society, Berkeley, CA, August 1981.

Clement, J. Students' preconceptions in introductory mechanics. *American Journal of Physics*, 1982, *50(1)*, 66–71.

diSessa, A. On learnable representations of knowledge: A meaning for the computational metaphor. In J. Lockhead & J. Clement (Eds.), *Cognitive Process Instruction*. Philadelphia, Pa.: Franklin Institute Press, 1979.

Drake, S. Galileo and the law of inertia. *American Journal of Physics*, 1964, *32*, 601.

Drake, S. Newton's apple and Galileo's dialogue. *Scientific American*, 1980, 243, 151.

Driver, R. P. *The representation of conceptual frameworks in young adolescent science students*. Unpublished doctoral disseration, University of Illinois, 1973.

Dyksterhuis, E. J. *The mechanization of the world picture*. Oxford: The Clarendon Press, 1961.

Franklin, A. Inertia in the middle ages. *The Physics Teacher*, 1978, *16*, 4.

Fredette, N. & Lochhead, J. Student conceptions of simple circuits. *The Physics Teacher*, 1980, *18*, 194.

Fuller, R. G., Karplus, R., & Lawson, A. Can physics develop reasoning?. *Physics Today*, 1977, *30(2)*, 23–28.

Fuller, R. *The ADAPT book*. Lincoln, Nebraska: ADAPT Program, University of Nebraska, 1977.

over again after point C." (See Appendix 1 for example of a Rocket problem transcript.) These interview results, and the consistent error pattern both within each problem and across the problems indicate that most errors are not due to random mistakes but rather are based on a stable misconception that is shared by many individuals.

DISCUSSION OF SIMILAR ARGUMENTS IN GALILEO'S WRITINGS

Two typical transcript excerpts from freshman engineering students working on the Coin problem are shown below:

Transcript of S1

S1: *So there's a force going up* and there is *the force of gravity pushing* it down. And *the gravity is less* because the coin is still going up until it gets to C. (Draws upward arrow labeled "force of the throw" and shorter downward arrow labeled "gravity" at point B in Fig. 14.2.)...if the dot goes up *the force of throw gets to be less and less because gravity is pulling down on it, pulling down.*

Interviewer: Okay, what about the length of this arrow ["force of the throw"]. If we use that to represent how strong the force is, would it be stronger than gravity at point B?

S1: Yeah, because the ball is still going up, so the force of the throw is still overcoming the force of the gravity that wants to make it go down.

Transcript of S2

S2: At B there'd be two—that I could think of. The upward force—the *upward original force* that was given to the coin to make it fly in the air...(Draws upward arrow at B). . .and the gravitational. (Draws downward arrow at B.) But the *reason that the coin is going up is because the original is greater than the gravitational.*

The italicized statements indicate that the students believe that a force is acting upwards on the coin at Point B, and that the coin is continuing to rise because the upward force is greater than the gravitational. This is evidence for the "motion implies a force" belief, in this case with reference to the sum of forces acting on the body.

TABLE 14.2
Types of Responses Given for Rocket Problem by
Freshman Engineering Majors Before Taking Physics

	n = 150 ENTERING FRESHMAN ENGINEERS			n = 43 AFTER MECHANICS COURSE		
1) (CORRECT) 	14	9%		8	19%	
2) (PARTIALLY CORRECT) 	40	27%		16	37%	
3) (RETURNS TO HORIZONTAL)  	62	41%	46%	9	21%	26%
4) (RETURNS PARTIALLY TO HORIZONTAL)  	8	5%		2	5%	
5) (OTHER)	26	17%		8	19%	

> of its speed go increasing, after its departure from rest, in that simple ratio with which the continuation of time increases [p. 159].

One of Galileo's strengths, in contrast to the philosophical generalists of his age, was that he was able to make deep progress by intentionally restricting his field of inquiry (in this case to kinematics.) But the quotations from Galileo indicate that real conceptual change in this area is an extremely difficult task which should not be underestimated.[3] The fact that Galileo propounded a careful and well-articulated impetus theory during part of his career, and the fact that present day students give explanations that are very similar in their basic aspects to that theory, is supporting evidence for the strong, intuitive attraction of the "motion implies force" belief. The students' errors appear not to be simply capricious; the belief appears to be an informal but plausible theory which has been constructed by students on the basis of experience. This historical comparison makes the high error rates for students on these problems somewhat less surprising.

SUMMARY OF CHARACTERISTICS FOR THE "MOTION IMPLIES A FORCE" PRECONCEPTION

By studying the error patterns discussed so far, we can summarize what appear to be the most common characteristics of the "motion implies a force" preconception:

1. Continuing motion, even at a constant velocity, can trigger an assumption of the presence of a force in the direction of motion which acts on the object to cause the motion.
2. Such invented forces are especially common in explanations of motion that continues in the face of an obvious opposing force. In this case the object is assumed to continue to move because the invented force is greater than the opposing force.
3. The subject may believe that such a force "dies out" or "builds up" to account for changes in an object's speed.

The diversity of situations in which this preconception surfaces suggests that it is a major source of the difficulties encountered by students in understanding the physical principles associated with the equation F = ma.

---

[3] Although there is wide agreement on the fact that Galileo never stated Newton's second law, the extent to which he progressed toward a statement of the first law of inertia has been a point of discussion. See Drake (1964, 1980); Losee (1966).

After these student explanations were analyzed we discovered that Galileo (1960) had made some similar arguments in his manuscript *De Motu* (*On Motion*). In explaining the motion of an object thrown upwards he states:

> The body moves upward, provided the impressed motive force is greater than the resisting weight. But that force, as has been shown, is continuously weakened; it will finally become so diminished that it will no longer overcome the weight of the body and will not impel the body beyond that point....As the impressed force characteristically continues to decrease, the weight of the body begins to be predominant, and consequently the body begins to fall....This is what I consider to be the true cause of the acceleration of motion [p. 89].

His explanation that "the impressed motive force is greater than the resisting weight" is similar in many ways to the students' explanations. S2 explains that the "upward original force...is greater than the gravitational," and S1 explains that the "force of the throw...is...overcoming the force of gravity." In fact, it is remarkable how similar the statements are, given the fact that the speakers are separated culturally by over 300 years. In each case, they describe a continuing upward force acting on the coin as a cause of motion, and state that the upward motion requires that this force be larger than the force of gravity.

Of course, Galileo thought much more deeply about these issues in his ingenious thought experiments than students do. When he published *Two New Sciences* much later in his career, Galileo (1974) presented essentially the aforementioned argument, but was unwilling to endorse or refute it.[2] He assigned the argument to Sagredo, the "middleman" in the dialogues, rather than to either Salviati, the spokesman representing himself, or to Simplicio, whose views are closest to Galileo's Aristotelian adversaries. Following Sagredo's presentation, in *Two New Sciences* (1974), Salviati says:

> The present does not seem to me to be an opportune time to enter into the investigation of the cause of the acceleration of natural motion...it suffices our Author that we understand him to want us to investigate and demonstrate some attributes of a motion so accelerated (whatever be the cause of its acceleration) that the momenta

---

[2] Sagredo:...it seems to me that a very appropriate answer can be deduced for the question agitated among philosophers as to the possible cause of acceleration of the natural motion of heavy bodies. For let us consider that in the heavy body hurled upwards, the force impressed upon it by the thrower is continually diminishing, and that this is the force that drives it upward as long as this remains greater than the contrary force of its heaviness...The diminutions of this alien impetus then continuing, and in consequence the advantage passing over to the side of the heaviness, descent commences...And since this [force] continues to diminish, and comes to be overpowered in ever greater ratio by the heaviness, the continual acceleration of the motion arises therefrom. (pp. 157–158)

IMPLICATIONS FOR INSTRUCTION

These findings lead us to suspect that it may be necessary to devote more attention to conceptual primitives at the qualitative level than is currently practiced, and that teaching strategies limited to expository presentation may be unlikely to succeed in this area. The "motion implies a force" preconception is not likely to disappear simply because students have been exposed to the standard view in their physics courses. More likely, Newtonian ideas will simply be misperceived or distorted by students so as to fit their existing preconceptions; or they may be memorized separately as formulas with little or no connection to fundamental qualitative concepts. Attempts to "cover" a very large syllabus, and to present physics primarily in a formal mathematical language, may preclude students from learning basic qualitative concepts that give them an intuitive understanding of the subject. Discouraging as these implications may seem, it should be remembered that historically, pre-Newtonian concepts of mechanics had a strong appeal, and scientists were at least as resistant to change as students are.

Serious difficulties with conceptual primitives have also been documented for undergraduates in several other areas of physics, including relative motion, torque (Barowy & Lochhead, 1980), simple circuits (Fredette & Lochhead, 1980), and acceleration (Trowbridge & McDermott, 1981). In addition, preconceptions producing consistent error patterns have been identified in the areas of Newton's Third law, centrifugal force (Viennot, 1979), velocity (Trowbridge & McDermott, 1980), elastic forces, and curvilinear motion (McCloskey, Caramazza & Green, 1980). These involve beliefs such as assuming that a stronger person experiences a smaller force than a weaker person when they push away from each other on an ice rink, drawing radially outward forces in circular motion, assuming that an object passing another moving object is travelling at the same speed when it is next to the object, believing that passive objects like tables cannot be sources of force, and believing that objects projected from a curved tube will continue to follow a curved path. Not all of these error patterns are as strong as the ones discussed here, but they do show up in a significant percentage of students.

Preconceptions need not be viewed exclusively as obstacles to learning, however. They constitute micro-theories that students have constructed on their own, and should be respected as such. Because they ordinarily have some predictive power in certain practical situations, they can be thought of as "zeroth-order models" which the students possess. Some preconceptions can be built upon or modified by students in order to increase the precision and generality of their theories.[4] In this approach the goal is to find teaching strategies that encourage

---

[4] Impetus theory, for example can be seen historically as an important intermediate step between Aristotle's antipersperis theory and the modern concept of inertia. For a discussion of how more formal physical principles may be connected to physical intuitions. see Clement (1979a).

POST COURSE RESULTS

In order to determine the effect of a physics course on these misconceptions we also tested two groups of students who had taken mechanics. The students in post group A were paid volunteers who agreed to take a diagnostic test before their final exam in a standard, one semester introductory mechanics course for engineers and science majors. Most of these students were sophomores and were from the same institution as the freshman group reported on earlier. The teacher of the course has received consistently high praise in written evaluations from students for his clarity of presentation, helpfulness, and genuine interest in teaching. The average grade in the course for these volunteers happened to be significantly higher than the course mean. The students in post group B were sophomore, junior, and senior engineering majors enrolled in an upper level engineering course at a second institution. All had previously taken mechanics.

Scores of the post course students were somewhat better than those of the prephysics students, but an alarmingly high number of students still gave wrong answers of the same kind on these very basic problems, as shown in Table 14.1. This was in spite of the fact that none of the problems required advanced mathematical skills. What they did require was adequate knowledge of the basic qualitative model for how forces affect motion.

On the Rocket problem, these students did somewhat better in avoiding the most blatant error: the misconception that the rocket will return to a horizontal path. However, on the Coin problem, the percentage of error only changed from 88% to 75% for group A, a rather disturbing result. In this problem, almost all errors were again in the form of an upward arrow. Additional data for this group show 44% drawing forces incorrectly on the Pendulum problem, with a 51% error rate at the second institution. Sixty-eight percent and 78% of these errors, respectively, included arrows drawn horizontally or tangential to the motion. Possibly, these error rates are lower than for the Coin problem because the opposition between the direction of motion and the gravitational force is more pronounced in the Coin problem. This is consistent with the fact that more invented forces were shown on the upswing of the pendulum than on the downswing.

Although the precourse and post-course tests were given to different groups, the two independent results indicate what can be expected of students before and after the introductory course, and the fact that post group A was an above average sample from the course leads us to be concerned about the level of understanding that is generally attained. In conclusion, the data support the hypothesis that for the majority of these students, the "motion implies a force" preconception was highly resistant to change. This conclusion applies to the extent that the students could not solve basic problems of this kind where the direction of motion does not coincide with the direction of the net force.

that sequences of analogies could be generated for fostering the understanding of other physical principles as well. Studies also indicate that at least some students have a useful set of physical intuitions that could be used as starting points for instruction (Clement, 1979a; 1979b). In addition, spontaneous hand motions produced by students in these studies indicate that "developed" physical intuitions underlying an understanding of such physical principles can involve a rather rich set of kinesthetic images. Further research is needed to determine how this mode of representation and the use of analogies might be tapped in instruction.

THEORETICAL IMPLICATIONS

Apparently one cannot consider the student's mind to be a "blank slate" in the area of force and motion. Many of the concepts presented in this area must displace or be remolded from stable intuitive concepts that the student has constructed over a number of years.

An important problem for future research is to determine the origin of the persistance of the "motion implies a force" preconception. Presumably the conception is rooted in everyday perceptual-motor experiences with pushing and pulling objects. We have seen that deeply seated mental models in the form of physical intuitions can be very compelling and resilient, even in the face of potentially contradictory evidence and/or earnest teaching. They have a power and "momentum" of their own quite unlike a memorized rule or a passive set of verbal propositions which would presumably be easy to "delete." They appear to have become "embedded into the system" at a perceptual-motor ("gut") level rather than at an abstract level.

However, it is not yet clear how we are to go about modeling the way such deep notions are represented mentally. The usual method is to represent them as lists of symbols in the form of rules or propositions; this leaves unanswered the question of the locus of meaning underlying each symbol. Such models fail to capture the representational richness that is suggested by the resistence of preconception to change, by subjects' references to "picturing," "imagining," and "feeling," by spontaneous hand motions simulating forces and movements, and by expressions of necessity and intuitive conviction during explanations (Clement, 1979a, 1979b, 1982). These phenomena may eventually be more fruitfully modeled in terms of visual and kinesthetic representations of a more analogue character.

A more general theoretical implication of these findings is that although various general reasoning skills are important in physics, domain-specific knowledge is also crucial. Knowledge structures that represent specific types of physical interactions must be structured in a particular way if they are to embody Newtonian concepts; but the preconceptions found in students are often strucstudents to articulate and become conscious of their own preconceptions by making predictions based on them. A second goal is then to encourage them to make explicit comparisons between these preconceptions, accepted scientific explanations, and convincing empirical observations. Similar strategies have been advocated by Fuller, Karplus, and Lawson 1977; Fuller, 1977, and Arons (1977) among others. In one attempt to develop this approach we are designing laboratory activities for introductory mechanics in which students are asked to give a large number of qualitative predictions and explanations about elementary phenomena such as the motion of the simple pendulum or of the tossed coin[5]. We have found that questions about the direction and relative magnitudes of forces, velocities, and accelerations at different points of the motion are quite challenging to introductory students. In the absence of formulas to "plug into," such questions are an effective way of getting students to think about their own preconceptions. In general, when qualitative misconceptions arise, it is necessary for students to express them and to actively work out their implications in order to see the advantages of the Newtonian point of view. Class discussions and arguments between pairs of students are especially helpful in this regard. Further development of innovative instruction techniques that emphasize the understanding of qualitative principles should be encouraged. Most importantly, instructional strategies that do not discourage students from making their own conjectures in the future should be sought.

Galileo was apparently aware of this type of teaching strategy, for his dialogues represent a marvelous attempt to deal directly with the common preconceptions and prevailing theories of his time at a qualitative level. The enormous conceptual breakthroughs that were achieved by Galileo were not easy to communicate to his peers. His writings appeal to the reader's intuitions by using concrete, practical situations to illustrate his theories (Galileo, 1960, 1962). He also took pains to present and discuss the Aristotelian theories he considered to be wrong, showing *why* he believed them to be wrong. One might do worse than to take these aspects of Galileo's teaching technique as a model for pedagogy today.

Another possible instructional strategy for teaching these concepts involves the use of analogies and kinesthetic intuitions. VanLehn and Brown (1979) have proposed an approach for teaching mathematical models underlying subtraction based on partial analogies between successively more complex models. The use of analogies to generate scientific models has been documented in thinking aloud interviews with scientists (Clement, 1981), and with students (Clement, 1979b). The latter study shows a college freshman spontaneously generating several analogies in a series of thought experiments in order to arrive at a fairly abstract understanding of Newton's third law and the concept of inertial mass. It may be

---

[5] Draft available from the author on request.

Galilei, G. *De motu*. Madison, Wisconsin: University of Wisconsin Press, 1960. Trans. by I. E. Drabkin.

Galilei, G. *Dialogue concerning the two chief world systems*. Berkeley: University of California Press, 1962. Trans. by S. Drake.

Galilei, G. *Two new sciences*. Madison: University of Wisconsin Press, 1974. Trans. by S. Drake.

Lawson, R. A., Trowbridge, D. E., & McDermott, L. C. Students' conceptions of dynamics. *American Association of Physics Teachers Announcer*, 1979, *9*, 87.

Losee, J., & Drake, S. Galileo and the law of inertia. *American Journal of Physics*, 1966, *34*, 430.

McCloskey, M., Caramazza, A., & Green, B. Curvilinear motion in the absence of external forces: Naive beliefs about the motion of objects. *Science*, 1980, *210*, 4474.

Trowbridge, D., & McDermott, L. An investigation of student understanding of the concept of velocity in one dimension. *American Journal of Physics*, 1980, *48*, 12.

Trowbridge, D., & McDermott, L. An investigation of student understanding of the concept of acceleration in one dimension. *American Journal of Physics*, 1981, *49*, 2242.

VanLehn, K., & Brown, J. S. *Planning nets*: A representation for formalizing analogies and semantic models of procedural skills. In R. Snow, P. Frederico, & W. Montague, (Eds.), *Aptitude learning and instruction: Cognitive process analyses*. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1979.

Viennot, L. Spontaneous reasoning in elementary dynamics. *European Journal of Science Education*, 1979, *1*, 205.