

Unity and variety in visual form

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Abstract. Some stimuli are perceived as unitary patterns, and others as dual or plural patterns. Such '**unity-and-variety**' phenomena are explained by various process approaches of perception, such as the global precedence hypothesis, the preattentive orientation detection assumption, and the **recognition-by-components** model. However, these three approaches, which will be discussed in this paper, each explain a different subset of these phenomena. It will be argued that not only these three subsets but also other **unity-and-variety** phenomena can be explained from just one point of view by adopting the descriptive minimum principle. This principle states that the preferred interpretation of a pattern is reflected by the simplest of all possible representations of that pattern. The highest **hierarchical** level in the simplest pattern-representation will be called the '**superstructure**' of the pattern. The superstructure of a pattern neither refers necessarily to the largest or global pattern component, nor is assessed **nessessarily** in a primary stage in the perception process. Yet, it will be argued that the superstructure is decisive in determining whether a shape is perceived either as unitary or as dual.

1 Introduction

Some stimuli can be perceived as singular patterns, others as multiple **patterns**. The question is: what causes perceptual unity and what determines variety in visual form? We begin with a demonstration.

Figures 1a and 1b both consist of the same components: circles, squares, and crosses. Yet, **figure 1a** is generally considered to be a unitary pattern, and **figure 1b** a dual pattern. The patterns seen by **squinting—the global shapes—seem** to be decisive; for **figure 1a** the global shape consists of a circle, and for **figure 1b** the global shape consists of a square and a cross. Assuming that these global shapes are **decisive**, various reasons can be given for the perceived unity and duality of **figures 1a** and **1b**. On the one hand, a circle is highly symmetrical, invariant under rotation (**Garner 1974**), its contour is closed and shows good continuation; these features unify the elements of the circle in **figure 1a**. On the other hand, in **figure 1b**, similar features may unify the elements of the square and, separately, the elements of the cross, but those features are absent in the combination of the square and the cross. Similarly, the flowers in **figure 1c** are unified by the relations between the flowers, not by **the** properties of the flowers themselves. An overall symmetry keeps them together (**Palmer 1983**). This is not true for **figure 1d**. The two different symmetries divide the latter figure into two parts. **Figure 1e** is unified by a repetition of subsequent pairs of flowers, whereas in **figure 1f** the grouping similarity merely holds within each part, not between the two parts. In each of the figures **1c**, **1d**, **1e**, and **1f**, the shapes of the flowers play a subordinate role; the unifying effects seem to be determined by overall properties.

The unity-and-variety phenomenon demonstrated in **figure 1** gives rise to two questions. The first and most important **question** is: are dominant pattern-properties always global and therefore larger than subordinate properties? Several authors have discussed this question (see **Koffka 1935**; **Hoffman 1975**; **Kinchla 1977**; **Navon 1977**; **Palmer 1977**; **Martin 1979**; **de Boer and Keuss 1981**; **Beck 1982**).

In this paper, special attention will be given to Navon's global precedence hypothesis and Beck's early orientation detection assumption. The second question is: by which properties can patterns and pattern differences be described? This question has also been studied by various scientists (see Hochberg and McAllister 1953; Attneave 1959; Garner 1962; Restle 1970; Simon 1972; van Tuyl 1980; Buffart et al 1981; Collard and Buffart 1983; Palmer 1983; Rock 1983; Leyton 1986; Pomerantz and Kubovy 1986; Treisman 1986; Biederman 1987). In this paper, the recognition-by-components model of Biederman will be discussed in more detail.

The models of Navon (1977), Beck (1982), and Biederman (1987) will be contrasted with the descriptive minimum principle approach. This principle stems from a Gestalt law, the law of *Prägnanz*, and has been formulated and quantified by Hochberg and McAllister (1953). The descriptive minimum principle states that the simplest of all representations of a pattern reflects the preferred interpretation of that pattern. This principle does not deal primarily with the process of perception but with the final pattern representations that are the outcome of the process (Hatfield and Epstein 1985). This, which for many scientists is a problematic aspect of the descriptive minimum principle approach, contrasts with the models of Navon (1977), Beck (1982), and Biederman (1987), which deal with the process of perception. However, by using the descriptive minimum principle approach, the answer to the first question (what are the dominant pattern-properties?) can be derived from the answer to the second question (how can patterns be described?). That is, the specification of dominant properties can be derived from simplest pattern representations. At least, this is what we will try to show. Furthermore, we will attempt to demonstrate that, while the models of Navon (1977), Beck (1982), and Biederman (1987) each explain different phenomena, these phenomena can all be explained by the descriptive minimum principle approach, ie from a single point of view. We will begin with our descriptive minimum principle approach since it is more efficient to discuss each phenomenon once **from two points of view** than twice **from one point of view**.

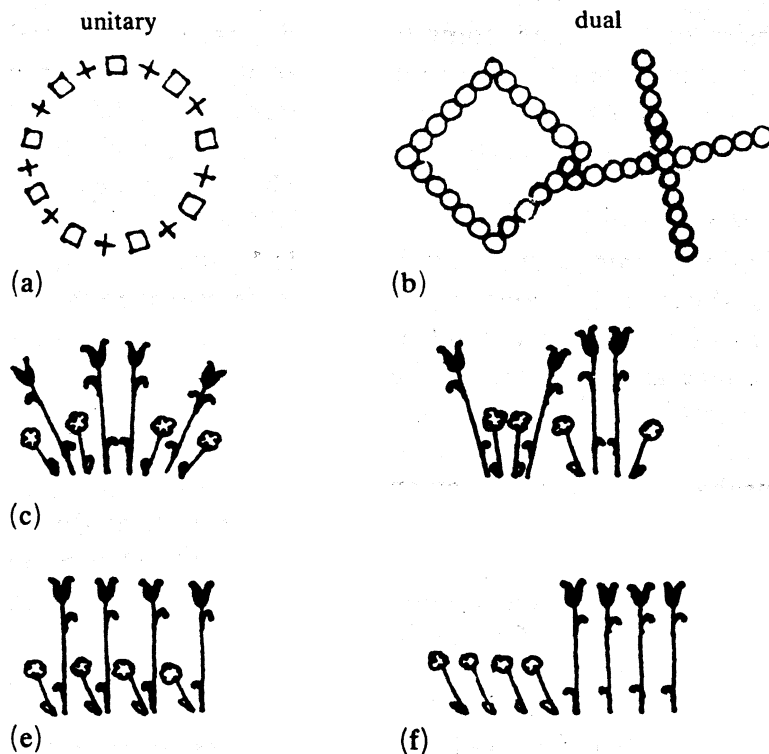


Figure 1. Demonstrations of unitary and dual patterns.

2 The **descriptive minimum principle** approach

2.1 *Gestalt laws*

Although the descriptive minimum principle (DM) approach has its origin in the Gestalt tradition, we begin to describe the DM approach by contrasting it to the alleged role of Gestalt laws in perception. These laws are supposed to specify the distal stimulus on the basis of proximal cues. The preferred interpretation of a distal stimulus is the one which is confirmed by most of the Gestalt laws applying to the stimulus. However, the information thus obtained is very partial and insufficient. Usually, this information merely concerns the organization of a pattern. Indeed, the knowledge that a certain stimulus consists of, say, three parts constrains the set of possible pattern representations but does not account for the fact that simple patterns can be remembered and recognized rather well. In other words, the Gestalt laws are not sufficient for representing patterns. These laws would be useful if the specific contribution of each law could be established. However, each law has no a priori fixed strength. Their strengths vary with each pattern by two factors, namely:

(i) The level of hierarchy in a pattern representation. As we will argue, the strengths of these laws are greater at a high level than at a low level. This implies that not just the separate Gestalt cues as they are present in patterns, but also the combinations of Gestalt cues should be known before the contribution of each Gestalt law can be established. In section 2.8, we will demonstrate this point.

(ii) The redundancy. The more redundant a Gestalt cue in a pattern, the greater its strength. This topic **will** be discussed in section 2.9.

2.2 *DM assumption*

The two factors mentioned above can be verified by perception research as we will show. In addition, a more theoretical argument can be given for the unreliability of Gestalt laws and local cues in general. There is no doubt that many laws reflect distal relations; for instance, the proximal projection of an object is smaller if it is at a greater distance. This fact establishes a proximal effect due to a distal cause. But local cues do the reverse. They derive distal properties from their proximal effects; for instance, the perceived object is supposed to be distant as its proximal image is small. However, this conclusion cannot be drawn from the above-mentioned fact and is at least unreliable. The DM approach makes no use of **effect-cause** information, but uses **cause-effect** knowledge during the creation of various distal shape-constructions whose projections agree with the stimulus. For instance, if the proximal stimulus is a black surface, at least two reconstructions of this stimulus are considered. The image may stem from a distal black-coloured surface or from a shadow. If it stems from a shadow, all other stimulus elements are interpreted as parts of a whole in which the shadow assumption fits. In this process, knowledge of projective geometry is useful. According to the DM approach each pattern **reconstruction—or distal pattern interpretation—is** represented by a 'code' and the simplest code is chosen (Leeuwenberg and Boselie 1988). Notice that a DM code describes the stimulus interpretation such that the stimulus interpretation can be reconstructed from its code.

2.3 *Questions*

So far we have dealt with the basic assumption of the DM model. This assumption primarily concerns the selection criterion for the preferred pattern interpretation and presupposes that distal pattern interpretations are somehow achieved. Although the latter issue is considered to be subordinate by the DM model, it should not be disregarded. Questions should be solved, such as: which kinds of regularity are involved in pattern descriptions (coding rules), how simplicity is defined (/ load), and how the simplest code is actually obtained. To a large extent, these questions have been

solved recently by van der Helm, but his analysis is too elaborate to present it here (van der Helm and Leeuwenberg 1991). We will merely give the gist of these issues, as follows.

2.4 Code syntax

The DM model presupposes that data can be described in a reduced form on the basis of the kinds of regularity present in the data. Hence, reduction rules are relevant. However, innumerable reduction rules and different kinds of regularities can be distinguished. Van der Helm (1989) has developed his so-called accessibility criterion for the selection of regularities that are appropriate in parallel information processing. Three kinds of regularities appear to satisfy his criterion, namely: iteration, symmetry, and alternation. These regularities are exemplified for a series of symbols, as follows:

patterns		codes	
a b a b	←	? (a b)	(iteration)
a b b a	←	S (a b)	(symmetry)
a b c b	←	⟨a c⟩⟨b⟩	(alternation)

The characters 'a', 'b' and 'c' are not meant to be part of the alphabet and do not obey the alphabetical order. They are symbols standing for pattern elements and even for whole pattern codes (hierarchical substitution). In these examples, the patterns are placed at the left and the codes at the right. This order agrees with the alleged processing order from pattern to code. However, the arrows point in the reverse order, from right to left. This order reflects the fact that only one pattern is described by a code. Inversely, as we will show later, one pattern can be described by several codes. The direction of the arrows also reflects the top-down character of the DM model.

2.5 Complexity

Besides reduction rules, a metric of structural information is needed. (Later on, we will be more explicit about the meaning of 'structural'.) Until 1990, we specified structural information load (I) as the number of symbols (like 'a', 'b', and 'c' in the example above) plus the number of iteration and symmetry operations in a code (not the alternation operations). The information units are indicated by the dots below the patterns and codes in the examples above, and were meant to reflect the amount of memory space needed to store a code. For all three patterns it holds that $I = 4$, and for all their codes $I = 3$. Recently, van der Helm has argued that this metric is theoretically inadequate and has introduced a new metric of structural information (van der Helm et al 1992). Introducing this new metric would go beyond the scope of the present paper since its foundation is rather difficult to explain, and it yields similar results to those of the old metric. Therefore, in this paper, we will stick to the old metric.

2.6 Code selection process

We will now discuss the process for selecting the simplest code of a pattern. For the moment we merely focus on patterns consisting of series of symbols and not on real visual patterns. This is because even for a series of symbols the minimum code selection is rather difficult to solve. Notice that a series of N symbols gives rise to about 2^N different codes, constructed by means of the three reduction rules mentioned earlier (van der Helm 1989). The assumption that during perception all these 2^N codes are actually constructed before the simplest is selected is unrealistic. It contradicts the rapidity of the perception process. An approach that accounts well for the rapidity of perception is one that presupposes that the process occurs in

equality of all line segments of the square and is reflected by the simplicity of the simplest code. Owing to the equality of line segments, the information load of the simplest square code, 4(1a), namely $7 = 3$, is less than that of the simplest rectangle code, $2(\langle 1m \rangle \langle a \rangle)$, namely $7 = 4$. The equality versus nonequality is a structural aspect. Thus, the simplest code of a square specifies a class of patterns with the same structure (Garner 1962; Collard and Buffart 1983). This class of squares is geometrically a subset of the class of rectangles.

So far, we have dealt with the second question, about pattern description, as indicated in the introduction. Now we will focus on the first question, about the dominance issue. We distinguish dominance determined by structural factors and dominance determined by metrical factors. In the following sections we mainly rely on intuitively judged complexities instead of on *I-load* measures of given pattern organizations. Note that these measures stem from a specific model of DM, whereas our goal is to argue in favour of the basic DM option. After all, partially correct quantifications of complexity in terms of *I-load* measures within a specific model do not necessarily falsify the basic DM option.

2.8 Structural dominance

The formula $10(A)$ may stand for 10 apples. The '10 times' part of the formula is considered as the qualifier and 'A' as the argument of the operation. The qualifier tells us something about the argument, but the reverse is not true. Because of this hierarchical relationship, we call the qualifier the 'superstructure' and the argument the 'subordinate structure'. Our hypothesis is that the superstructure dominates the subordinate structure in the perception of patterns. We will attempt to show this. Furthermore, we will demonstrate that the mentioned hierarchical relationship is not based on the supposition that the superstructure always refers to a wholistic or global aspect and the subordinate structure to a local part of a pattern. First of all, notice that the hierarchical relationship between the superstructure and the subordinate structure is not necessarily related to any temporal order in the perception process: whether one establishes first 10 somethings and later that each something is an apple, or whether one first sees one apple and later that there are 10 of them, does not change the super-subordinate hierarchy in $10(A)$. Yet, the hierarchical super-subordinate relationship is asymmetrical. That is, for instance, reversing the relationship between 10 and A, as given by $10(A)$, into $A(10)$ makes no sense, since $A(10)$ has no meaning. As a further illustration, consider figure 2; the flowers represent the lowest hierarchical level and, in both figures 2a and 2b, symmetry and iteration represent subsequent higher hierarchical levels in the pattern codes. In the code in figure 2a, the iteration represents the superstructure and the symmetry the subordinate structure, whereas in figure 2b the code shows the reverse relationship in that the symmetry represents the superstructure and the iteration the subordinate structure. So, in this case, reversing the hierarchical relationship of components in a

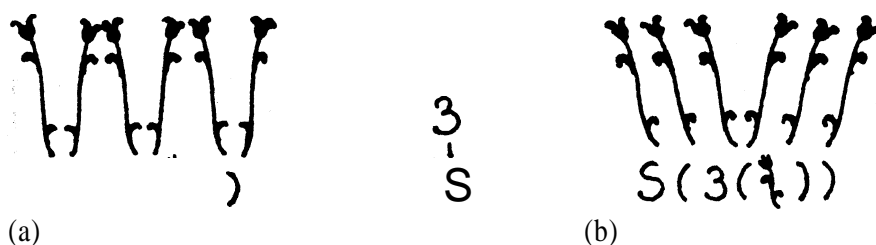


Figure 2. Two hierarchically opposed flower configurations. In the code in (a) the iteration is the superstructure and the symmetry is the subordinate structure. In the code in (b), the symmetry is the superstructure and the iteration is the subordinate structure.

code does yield a meaningful code, but the asymmetry of that **relationship** is clearly illustrated by the fact that the two codes represent different patterns.

In order to approach the above questions about the dominant and the global properties of the superstructure, we will consider those cases in which reversing the hierarchical relationship of components in a code yields a meaningful code, so as to get a balanced comparison. Note that, at the right-hand side in figure 2, the hierarchical **super-subordinate** relationship between the code components has been depicted in a simplified vertical way. In the remaining part of this paper, we will only use this simplified way and, moreover, we will focus on patterns for which these code components can easily be visualized in terms of the patterns themselves, in order to provide a better view for testing our superstructure-dominance hypothesis.

We will first illustrate several aspects of the **super-subordinate** relationship, by means of figures 3, 4, and 5. The pattern in figure 3a can be described in innumerable ways. Only two codes are considered as they comprise plausible components. The two codes are visualized in figures 3b and 3c. Without formal analysis, it will be clear that the first code is simpler than the second one. According to the minimum

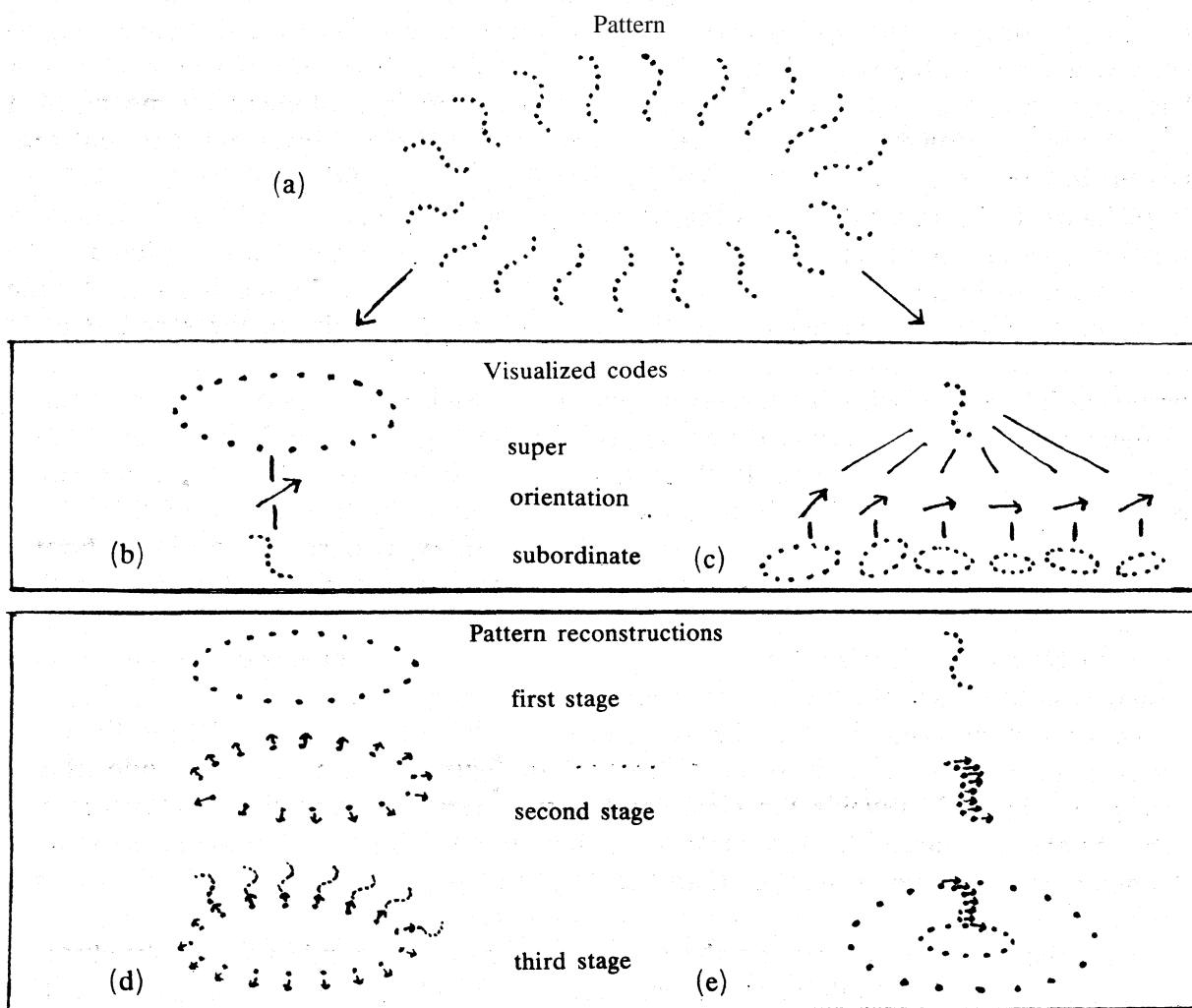


Figure 3. The pattern (a) can be described in innumerable ways. The visualized simplest code (b) reveals the hierarchy: ellipse, angle, S shape. The ellipse refers to the superstructure, being the independent code component. The S shape refers to the subordinate code component, being dependent on the superstructure. An alternative code, shown in (c), with a hierarchy reversed to that in (b), is more complex than the latter. The reconstructions shown in (d) and (e) from the codes in (b) and (c), respectively, follow the hierarchical levels of the codes in a stepwise manner.

principle, the first code is selected and preferred, if it is the simplest of all possible codes. Since here we want to focus on the differences between the two codes, let us assume that the first code is indeed the simplest one.

The meaning of the codes can be clarified best by showing the way they prescribe the pattern reconstruction. The reconstructions of both codes are shown in figures 3d and e. Each reconstruction occurs in three stages, which correspond to the three hierarchical levels in the codes. The code in figure 3b prescribes, first, the construction of a dotted ellipse corresponding to the superstructure in the code. In the second stage, orientations are attached at each dot of the ellipse. These orientations are constant with respect to the contour of the ellipse. In the third stage, each orientation is prolonged by means of a dotted S shape, which corresponds to the subordinate structure in the code. As is clear, the code itself indicates the reconstruction components in a very concise way. Again, notice that the sequential stages of the pattern reconstruction from the code do **not** imply such sequential stages in the process from the pattern to its code! According to the DM model the code is an a posteriori outcome of a selection process and not of a priori fixed stages.

The code in figure 3c shows a hierarchy which is the reversal of the hierarchy in the code shown in figure 3b. It prescribes a pattern reconstruction which is rather difficult to imagine and which involves, first, the construction of a dotted S-shaped superstructure. In the second stage, to each dot of the S shape, a different orientation has to be related. In the third stage, each orientation is prolonged by means of a dotted ellipse. However, just like the orientations, all these ellipses are different too, namely in size.

In figure 4 and in all the following figures, the codes are **shown** without orientation relations, ie only the first and third hierarchical-level components are depicted. The pattern shown in figure 4a, which is the same as that in figure 3a, has a simplest code in which the hierarchy is reversed with respect to the hierarchy in the simplest code of the pattern in figure 4b. Note that these codes represent completely different patterns. The same holds for figures 4c and d, in which the codes are similar to those in figures 4a and b. The only difference is that the angles are in 3-D instead of 2-D (Leeuwenberg 1971a, 1971b). In the same way as illustrated by figure 3c, one could represent the shape in figure 4c by a code with a hierarchy the reverse of that of the simplest code. However, such a code is rather complex; it describes a pile of hoops, each with a different size. For the shape in figure 4d, the reversed code is rather complex too; it describes a circle of aligned **S-shaped** wires. These **S-shaped** wires seem to be similar, but in fact are all different! The difference between the upper **S-shaped** contour and the lower **S-shaped** contour is the most evident.

An important point is that the superstructure is not necessarily larger than the subordinate structure. This is demonstrated in figure 5. The simplest code of the pattern in figure 5a reveals the circle as the superstructure and the S pattern as the subordinate structure. Yet, the circle is smaller than the S pattern in the sense that its diameter or perimeter is smaller than the length of the S pattern. Figure 5b shows a similar case.

A heuristic **trick** to determine the superstructure and the **subordinate** structure as given by the simplest code is to reconstruct the object by moving a rigid wire. For instance, moving a rigid **S-shaped** wire about a straight **axis** produces the shape shown in figure 5a. The rigid shape always refers to the subordinate structure, and the motion pattern to the superstructure. The shape in figure 4d can be suggested by moving a rigid circle along an S shape.

Now that we have illustrated several aspects of the **super-subordinate** relationship, **we** will focus on unity and variety of patterns. We hypothesize that the superstructure dominates subordinate structures, as the superstructure refers to a least nested

pattern property on which a subordinate structure depends. An indication of our hypothesis is given in figure 6, showing three shapes with their simplest codes. The codes of the patterns shown in figures 6a and 6c comprise single superstructures, whereas the code of figure 6b has two superstructures. In our view, the patterns in figures 6a and 6c are conceived as unitary shapes, whereas that in figure 6b is conceived as an odd dual shape.

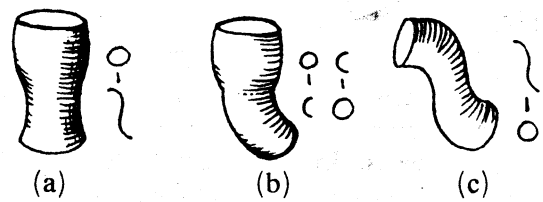
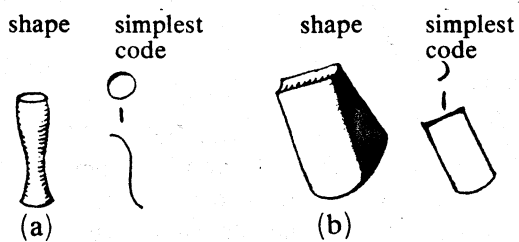
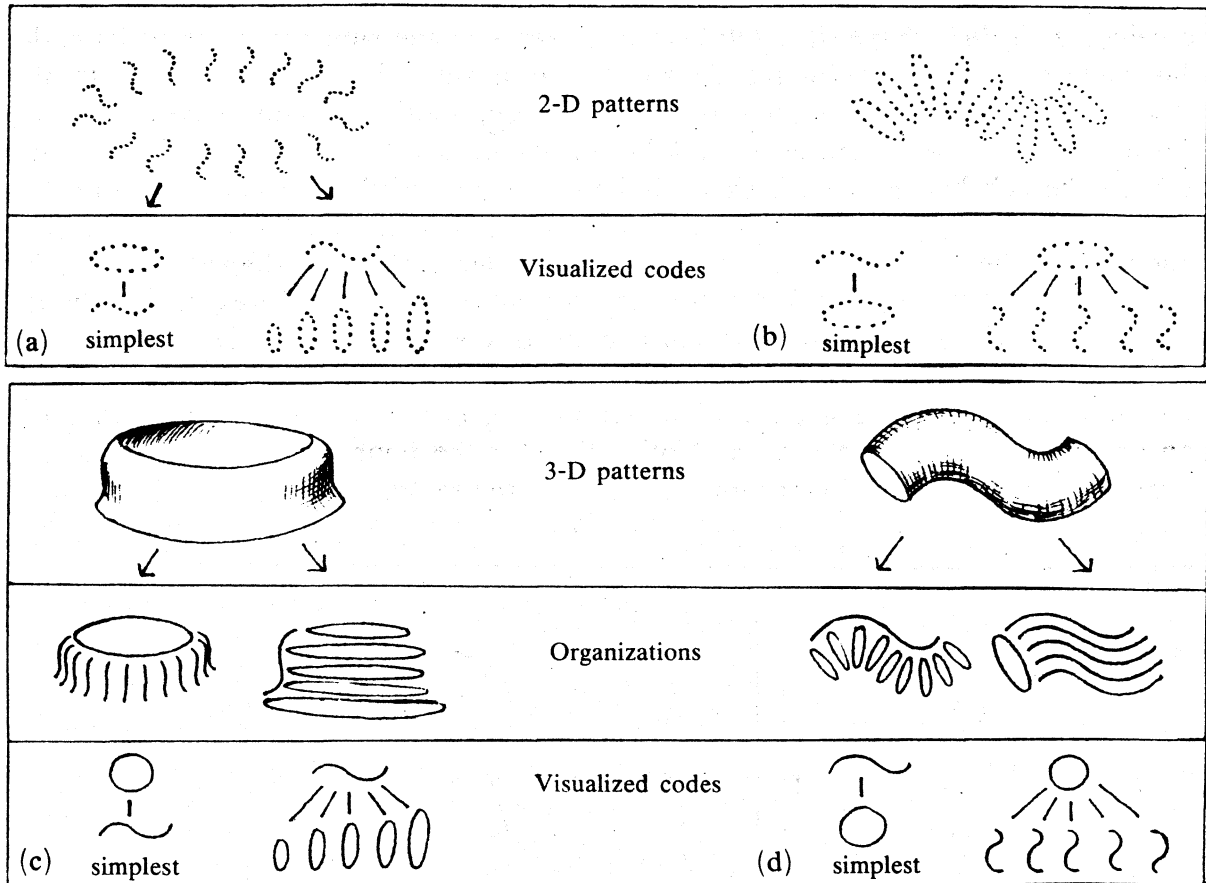


Figure 5. Two cases in which the superstructures of the simplest codes refer to pattern parts that are smaller than the parts to which the subordinate structures refer.

Figure 6. The simplest codes of the patterns shown in (a) and (c) comprise single superstructures, whereas the simplest code of that in (b) has two superstructures. The superstructure-dominance hypothesis predicts that those in (a) and (c) are conceived as unitary shapes, and that in (b) as a dual shape.

In section 3, evidence for our dominance hypothesis will be provided when commenting on the various process models. Here we will briefly report an experiment, performed recently by van Bakel (1989) in our laboratory. One hundred and forty-four pairs of computer-drawn shapes were presented to each of thirty subjects. The task was to indicate which shape of a pair was perceived as a unitary shape as opposed to a dual shape. In fact, each shape itself was subdivided into two parts. In figure 7 a balanced subset of four shapes, out of the nine experimental subsets, is presented together with their visualized codes. These codes were of course not shown to the subjects in the experiment. According to our dominance hypothesis, a shape is perceived as a dual shape, if a subdivision appears in the superstructure of its code. This applies to those in figures 7a and 7c. If a subdivision appears only in the subordinate structure of the code, the shape is supposed to be conceived as unitary. This applies to those in figures 7b and 7d. In one subset of the patterns, the distance between the subshapes of each shape is kept constant while, in another subset, the inferred volume between these subshapes is kept constant. The sizes of the superstructure and the subordinate structures are controlled too: in the shapes in figures 7a and 7b, the superstructures are smaller than the subordinate structures; in the shapes in figures 7c and 7d, the superstructure coincides with the larger structure, ie the global structure. The results are highly significant and support, almost without exception, the dominance hypothesis under varying conditions with respect to position and orientation of the shapes. Notice that for the figures 7c and 7d a reversed hierarchy in the codes is as inappropriate as it is for figure 4d.

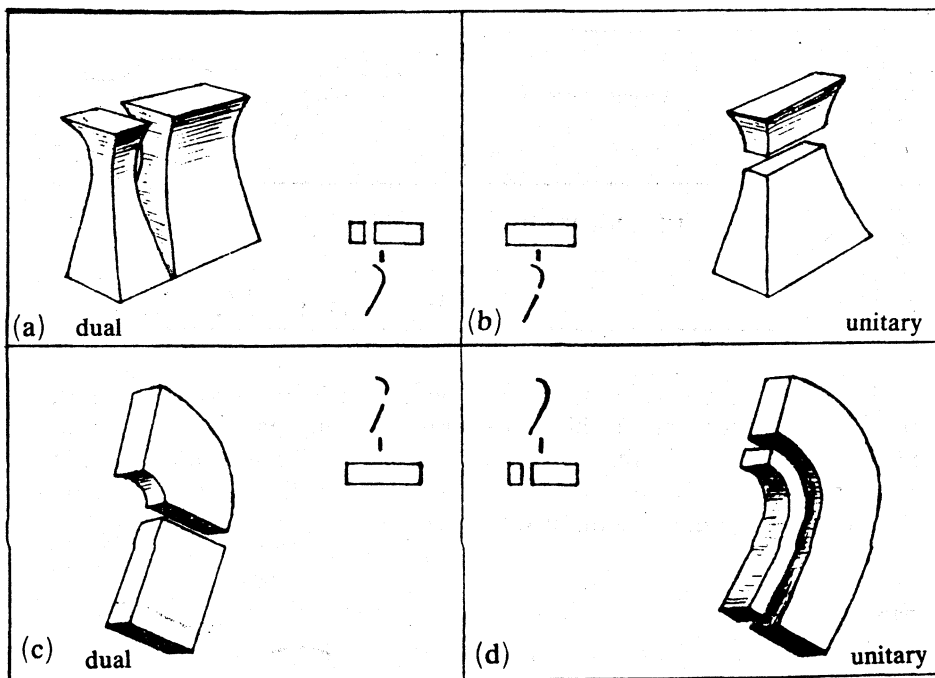


Figure 7. One of the *nine* sets of four patterns used in the experiment by van Bakel (1989). The superstructures in (a) and (c), as well as the subordinate structures in (b) and (d), comprise disjunct parts. In agreement with the superstructure-dominance assumption, the shapes in (a) and (c) are conceived as dual and those in (b) and (d) as unitary. Notice that the superstructures in (a) and (b) refer to smaller pattern components, and those in (c) and (d) to larger pattern components.

2.9 Metrical dominance

There are patterns that seem coherent **but** that cannot be described by a single code. Such a pattern has to be represented by two or more codes which complement each other. An example is given in figure 8a. This pattern, A, is conceived as consisting of

two 3-D components, namely A' and A'' . These components are incompatible in a single 3-D space. The suggested coherence is due to an external factor, such as the specific point of view from which the two components are seen, or to the accidental connection between the projections of the two components (Hochberg 1982; Rock 1983; Boselie and Leeuwenberg 1986). The question arises as to whether figure 8b has to be described by two complementary codes too. This pattern seems more coherent than that in figure 8a, and is similar to most patterns we have dealt with so far; it consists of a global shape, and of subpatterns the positions of which are determined by the global circular shape. The main incoherent aspect of the pattern in figure 8b is the fact that the orientations of the subpatterns are not related to the global circular shape. The subpattern orientations seem to be related to a frame external to the global shape. Such orientations we call 'absolute' as opposed to 'relative'.

From an ecological point of view, there is an argument for the assumption that the pattern in figure 8b is a combination of unrelated components. This figure reminds one of a joy wheel, in which case the orientation of the cars is determined by gravity. The orientation cannot be derived from a viewpoint-independent description of the global wheel or of the cars. In reality, this lacking relationship is guaranteed by the nearly frictionless connection between the cars and the wheel.

Outside the domain of ecology, there is an indication that a configuration of parallel-oriented subpatterns is incoherent; figure 9a has a square global structure and an S-shaped substructure. If one decreases the size of the square and increases the size of the S-shape, the pattern in figure 9a gradually changes into that in figure 9b and finally into that in figure 9c. However, the latter figure has a reversed phenomenal organization. Its global structure is S-shaped and its substructures are squares. The question is whether the structural organization of the pattern in figure 9c is also reversed with respect to that in figure 9a. To answer this we make use of the distinction between structural and metrical information, made by Gabor (1946) and MacKay (1950, 1969); structural information refers to dimensions, or 'logons', and is determined by identity relationships. Metrical information refers to quantitative variations and contributes to the redundancy—or 'weight of evidence' per unit—of structural information. According to these concepts, figures 9a and 9c do not have different structures and, hence, do not comprise a hierarchically dominant component in a

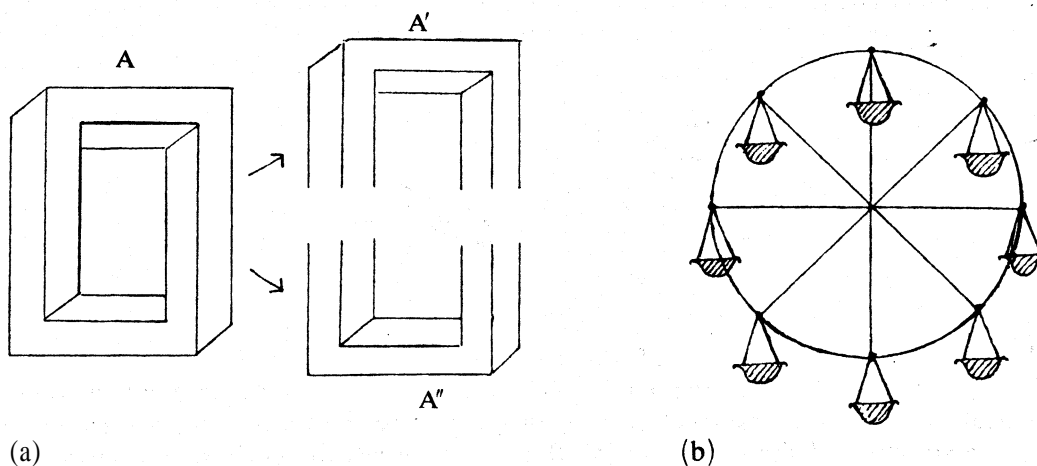


Figure 8. Examples of incoherent patterns. (a) Pattern A consists of two components which are incompatible in 3-D space. Therefore A cannot be described by a single code. The apparent coherence of A is merely due to a coincidental relationship between the projections of the two components. (b) In this pattern the orientation of the parts is not related to the global structure, but to some external frame.

structural sense. The dominant components in these two patterns are obviously due to metrical aspects. For instance, the square component in figure 9a is dominant as the small S-shaped substructures do not distort the square shape. In other words, the square is clear. This is less clearly the case in figures 9b and 9c. Beside the metrical effect of size (Goldmeier 1972; Martin 1979; Kimchi and Palmer 1982), there is the metrical effect of amount. The square shape is less well supported by figure 9d than by figure 9a, as the square is less redundantly represented in figure 9d. The weak support for the square holds not only for figure 9d but also for figures 9e and 9f. Similarly, in the 3-D figures 9g, 9h, and 9j, merely metrical factors determine the dominant component.

We have shown that the patterns in figure 9 lack a definite structural hierarchy. Now, the question is whether this lack is due to two or more equally simple complete codes with different structural hierarchies (which render a pattern structurally ambiguous), or due to two or more incomplete codes as holds for the 'impossible' shape in figure 8a. Our answer is that both alternatives are true, but for different patterns. In figure 10, A is a pattern which can be described by two complete codes with opposite structural hierarchies. According to one code, A', the superstructure is a straight line. This superstructure allows parallel orientations to be described by means of relative angles. There is another situation in which parallel orientations can be coded by means of relative angles. This situation applies especially to 3-D shapes

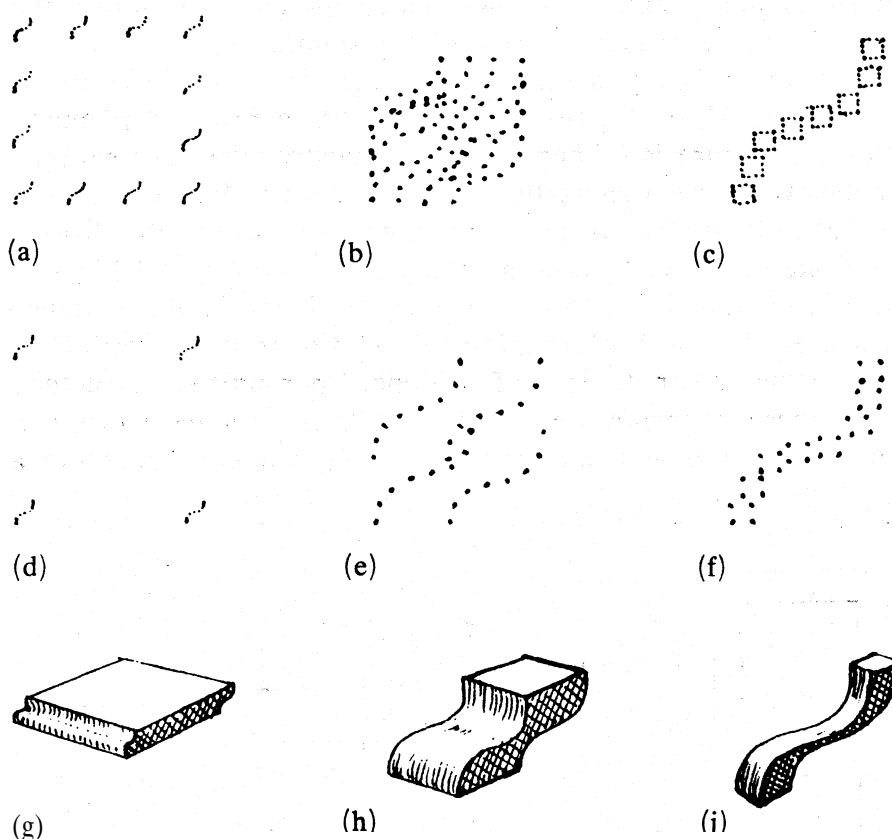


Figure 9. In all patterns, the orientations of the subpatterns are in parallel and therefore independent of the global shape. This property implies that the three patterns in a row *merely* differ in a metrical respect; for instance, if one increases the S-shaped subpatterns and decreases the square structure of the pattern in (a), this pattern gradually changes into those in (b) and (c). The same holds for the patterns in (d), (e), and (f), and in (g), (h), and (j). The patterns in a row therefore have the same structure. Hence, the dominance of the square in (a), and the dominance of the S shape in (c) are merely based on metrical qualities.

and is illustrated by another code, A". Its subordinate structures are straight lines being orthogonally oriented with respect to the plane of the superstructure. According to this code, A is conceived as a 3-D ribbon. As A allows two complete equally complex codes with different hierarchies, this figure has an ambiguous hierarchy.

However, only in the above-mentioned two situations can parallel orientations be coded by means of relative angles. Many patterns, such as A in figure 8 and the ones in figure 9, cannot be coded by means of relative angles in a simple way. These patterns might be described in a simple way by means of absolute angles. In that case, these patterns would be structurally ambiguous just like A in figure 10. However, as we have stated earlier, codes using absolute angles do not represent viewpoint-independent structures of the patterns. This is an argument against the use of absolute angles for describing parallel orientations. In support of this argument are the odd effects which are obtained if one does accept the use of absolute angles in codes. In that case, the patterns B, C, and D in figure 10 all have the same code; the superstructure is an S shape and the subordinate structures are parallel-oriented triangles. The differences between these three figures are merely due to slightly different orientations of the triangles. Notwithstanding the common code, the three figures are hardly conceived as patterns of the same class. The patterns C and D are obviously more complex than B and do not reveal the S component and the triangle components clearly. Moreover, pattern B in isolation is hardly interpreted according to its absolute-angle code. This code implies that the dark side of pattern B stands for a flat surface with a varying width, whereas this side is conceived rather as a 3-D curved surface of an object with a constant cross section. However, this object can be coded properly by means of relative angles. We conclude that absolute angles are not permitted for describing parallel orientations.

Finally, we illustrate some nonhierarchical relations within single complete-pattern codes. Such codes combine internal pattern structures and external space structures. There is no hierarchical relationship between these structures. Pattern A' in figure 11 is coded as a regular cross, shown as pattern A, placed on the surface of a cylinder. A technical account for the junction between such internal and external structures has been proposed by Leeuwenberg (1971a, 1971b). In a similar sense, shape B' is a regular 3-D cross, shown as B, which is deformed by the cylindrical space.

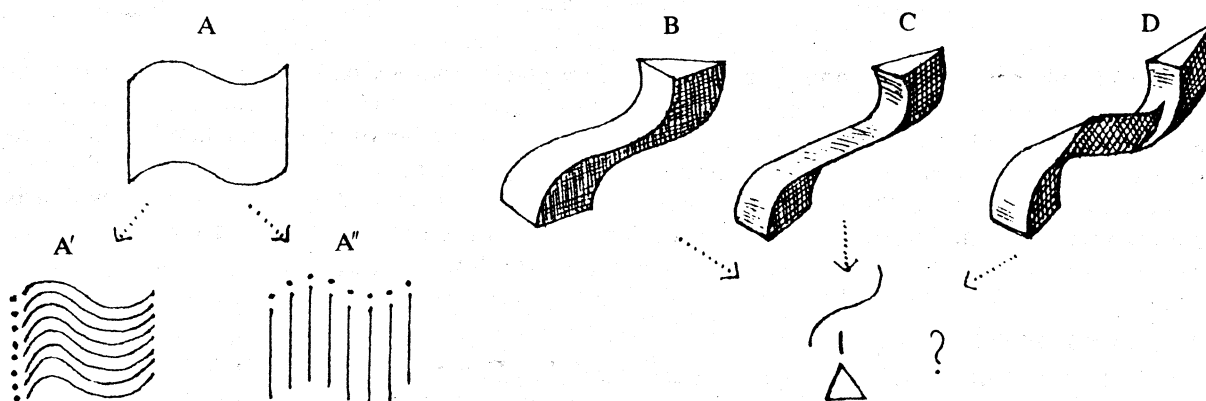


Figure 10. Patterns A, B, C, and D all lack a definite structural hierarchy, but for different reasons. Pattern A can have two complete codes, A' and A". Each code describes parallel orientations by means of relative angles. As the hierarchy of A' is the opposite of that of A", pattern A is hierarchically ambiguous. This is not true for patterns B, C, and D. The parallel orientations of the triangle subpatterns cannot be described simply by means of relative angles, but can be described simply by means of absolute angles. However, if these absolute angles are permitted in codes, patterns B, C, and D should belong to the same class and should be equally complex. The fact that this is clearly not true supports the assumption that parallel orientations should not be described by absolute angles.

Notice that within this space, shape **B'** is a regular cross, as shape **B**. Shape **C'** is a slightly deformed cone, shown as shape **C**. Shape **D'** is a strongly deformed cone, shown as **D**. The two cone deformations caused by the external space differ in an angular metrical respect. Moreover, the deformed objects themselves, shapes **C** and **D**, differ in an angular metrical respect. Shape **C** is more a prototypical cone than is shape **D**. The latter figure even resembles a tube. Both the metrical aspects of the external structure and those of the internal structure seem to determine the dominant components of the deformed objects; in shape **C'** the cross section seems more dominant than the axis. In shape **D'** the curved axis seems more dominant than the cross section. This perceptual effect agrees with the above-mentioned non-hierarchical relationship between the internal and the external structures in codes which combine both structures.

2.10 Summary

Unity, as opposed to variety, of patterns is specified by the superstructure, not by the subordinate structure. Both structures are components of simplest pattern codes, instead of direct pattern features. The **super-subordinate** hierarchy is based on an **operation-argument** relationship in the code. The hierarchy may indeed prescribe an order of stages in the reconstruction of the pattern from its code, but not in the process from pattern to its code. The superstructure, which is supposed to **dominate** the subordinate structure, does not necessarily coincide with the largest and, hence, with the global structure. Metrical aspects of a pattern may still **codetermine** dominance effects, but mainly in case of a missing structural hierarchy.

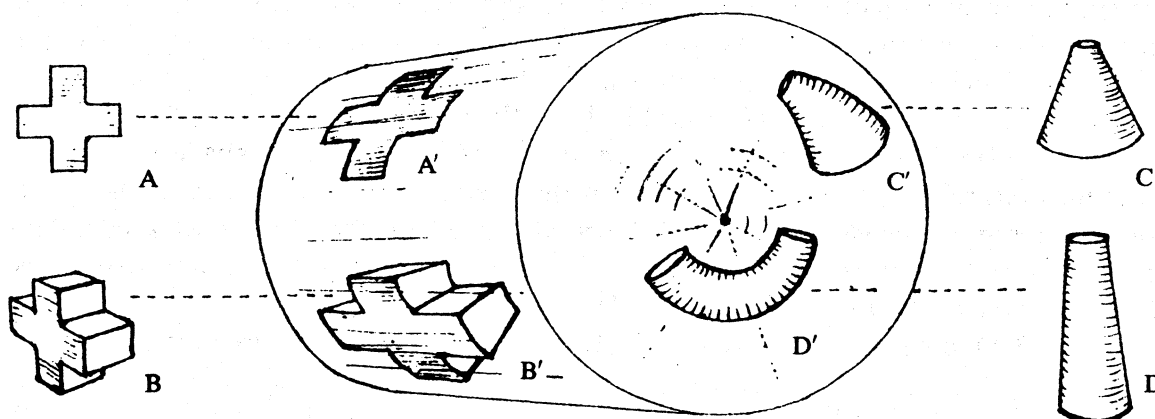


Figure 11. Shapes **A'**, **B'**, **C**, and **D'** are obtained by placing shapes **A**, **B**, **C**, and **D** in a cylindrical space. However, there is no hierarchical relation between the internal structures of **A**, **B**, **C**, and **D** on the one hand, and the external structure of the cylinder on the other hand. The dominance of a pattern component is determined by the metrical supremacy of one of the two structures. For instance the dominant component of **C'** is its expanding cross section, which is mainly related to the internal structure of **C**. The dominant component of **D'** is its circular axis, which is mainly related to the external structure of the cylinder.

3 Process models

We will now comment on some process models. For these models it is essential that the perceptual organization is ascribed to characteristics of the perceptual process, whereas the minimum principle derives this organization from pattern representations. It is less essential that these models assume subsequent process stages, yet we will treat them as straightforward linear stage models, even though they may not be meant to be treated as such by the authors. We take this liberty, since otherwise no clearcut predictions about pattern **classification** can be derived to be compared with the ones derived from the minimum principle.

3.1 Global precedence

Usually, an architect or painter begins with a global sketch and ends with the elaboration of details. These artists transpose concepts into patterns. Navon (1977, 1981) has attributed an analogous strategy to the reverse process, from patterns to concepts, namely perception: the extent of a pattern which is seen by squinting, ie the global shape, is supposed to be processed before the details. Navon (1977), as well as Hoffman (1975), have found support for this assumption from classifications of texture-alike patterns, such as those shown in figure 12a. The preferred subdivision of this series is (A, A') and (A''), rather than (A) and (A', A''). The preferred subdivision agrees with the common global shapes, instead of with the common details, even after prolonged inspection. The global precedence hypothesis is criticized by Pomerantz and Sager (1975), Kinchla (1977), Kinchla and Wolfe (1979), and Martin (1979). At least, they have indicated restrictions of this model. The concept of global shape, defined as low-spatial-frequency structure, is rather inconsistent with the experimental findings of Palmer (1977). Furthermore, de Boer and Keuss (1981) have provided arguments for the assumption that the preferred global shape is a post-perceptual dominance effect, instead of a perceptual precedence effect. In other words, the global preference is attributed to a characteristic of a pattern representation, and not to an effect of the process from pattern to its representation. We tend to agree with them. At least, the classifications of the texture-unlike patterns shown in figures 12b and 12c can be reconciled with their view while it is not a consequence of the global precedence hypothesis. Although the first two patterns, B and B', as well as C and C', share their global skeletons, the last two patterns of each series, B' and B'', as well as C' and C'', are grouped together, at least after prolonged inspection. If the global precedence hypothesis only applies to texture-alike patterns, a stage needs to precede the processing of the global shape, in which it is established whether or not stimuli are texture-alike patterns.

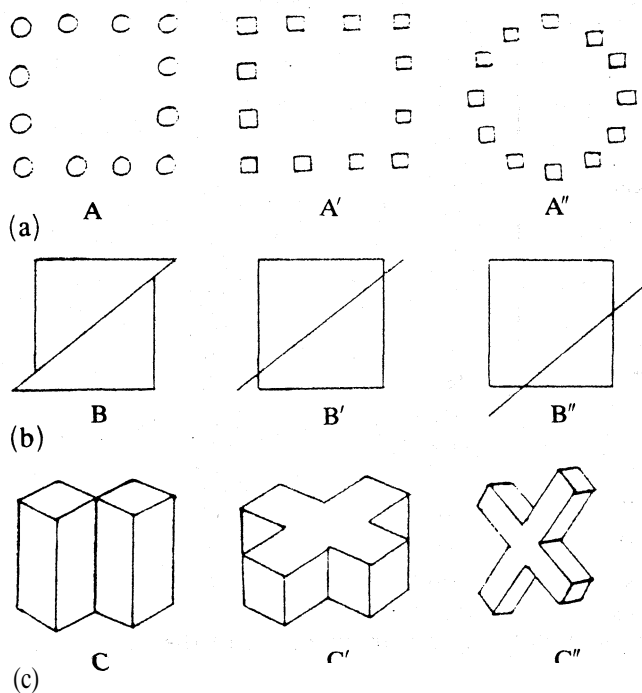


Figure 12. The first two patterns (A and A') share a common global shape and are perceptually grouped together in accordance with Navon's global precedence hypothesis. However, the perceptual organization of the second and third series (B, B', B'' and C, C', C'') disagrees with this hypothesis: the first two patterns in each series (B, B' and C, C') share a common global shape, whereas the latter two patterns (B', B'' and C, C'') are perceived as being the most similar patterns. The perceptual organizations of the three series all follow from the simplest pattern codes.

The minimum principle discussed in the previous section is also in line with the assumption of de Boer and Keuss (1981). According to this principle, patterns A and A' in figure 12a share the superstructures of their simplest codes. These superstructures refer to the positions of the subpatterns, and thus happen to coincide with the global shapes. The superstructures are supposed to be dominant with respect to the subordinate structures. Hence, this approach predicts the preferred grouping (A, A') and (A'') of figure 12a as well as does the global precedence hypothesis.

3.2 Preattentive orientation detection

The patterns in figure 13b comprise unseparated configurations of randomly positioned upright Ts, tilted Ts, and tilted Ls. Beck (1982) showed that such a configuration is, at first glance, organized in groups of subpatterns with the same orientations, instead of the same shape; thus (B) and (B', B'') instead of (B, B') and (B''). Beck assumed that in a prefocal stage of perception, texture-alike configurations are processed by orientation detectors. He showed furthermore that this organization does not hold for three singular characters: one upright T, one tilted T, and one tilted L. In fact, these are presented in figure 13a, disregarding the subpatterns. The three singular characters are organized on the basis of shape. As is clear from the preferred grouping, (A, A') and (A''), in figure 13a, the organization based on orientation is rather overruled by the organization of their global shapes. Hence, the prefocal orientation detection ought to be preceded by a very primary stage in which it is established that the stimulus is a random configuration of many subpatterns.

The organization of the patterns in figure 13 can also be explained as the postperceptual dominance effects of their simplest codes. The organization (A, A') and (A'') in figure 13a can be attributed to the dominance of their superstructures.

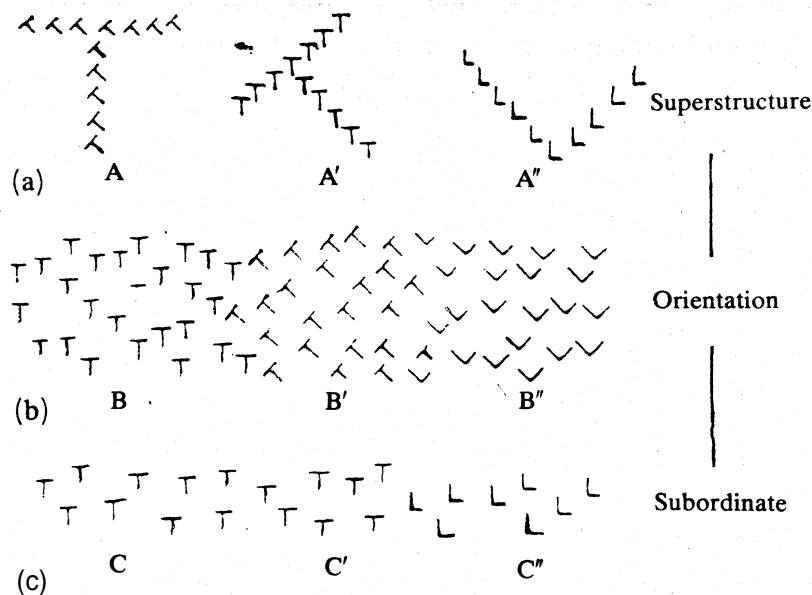


Figure 13. The two left-hand patterns in the first row are grouped together. This organization agrees with Navon's global precedence hypothesis. The patterns B' and B'' are perceptually taken together. This organization is, according to Beck, due to orientation detectors, which are supposed to be sensitive in a preattentive stage of the perception of homogeneous scattered subpatterns. According to the minimum principle approach, the organization of the series in (a) as (A, A') and (A'') is due to the dominance of the superstructure. This structure refers to the positions of the subpatterns. The organization of the series in (b) is attributed to the dominance of the highest hierarchically distinctive level. This level refers to the orientation of the subpatterns. The actual superstructure is in fact indecisive. For the series in (c) only the lowest level is decisive.

They correspond to the shapes of the three characters. Notice that these superstructures are not, in their turn, subordinate structures of a higher-order superstructure of three aligned positions, because then the 'subordinate structures' would all be different. In that case, a hierarchical code has no special economic advantage but even is at a disadvantage.

The superstructure of the patterns B, B', and B'' in figure 13b consists of the positions of the characters. However, these are randomly scattered in a single homogeneous cloud. As this superstructure does not provide any cue for a specific organization, it is assumed that refuge is taken at the highest distinctive hierarchical level. This level is specified by the orientation of subpatterns (see previous section). If this second level is undecisive too, the organization relies, of necessity, on the lowest level of hierarchy, specified by the shapes of the subpatterns (see figure 13c).

3.3 *Recognition by components*

A huge variety of shapes can be constructed from a few primitive elements. This happens in nature as well as in art. A few atoms can be combined into many different molecules. Three basic colours suffice to make many different pointillistic paintings. The knowledge of basic elements may also help to classify complex combinations of these elements. The recognition of words seems to be an illustration of this principle. All words can be constructed by means of about forty phonemes. The fact that each individual phoneme can be identified in a word, allows thousands of words to be identified and recognized as permutations of phonemes. Biederman (1987) has proposed the so-called **recognition-by-components (RBC)** model for the early identification of objects in analogy to word recognition by means of phonemes. He distinguishes thirty-six simple 3-D shapes, called 'geons', and argues that each of them can be identified from its retinal projections. His model implies that these components are identified in a primary bottom-up stage of **perception, which** precedes the assessment of the relations between components or the further specification of smaller components within a geon. In this sense, this approach can be considered as a stage model. We have selected Biederman's perception model for discussion, as it is, in our view, a prototypical approach of the perception process, integrating various ideas of other scientists (Garner 1962; Winston 1975; Navon 1977; Barrow and Tenenbaum 1982; Binford 1981; Ballard and Brown 1982; Marr 1982; Hoffman and Richards 1985).

Biederman (1987) provides experimental evidence for his thesis, **showing** that by geon decomposition, complex objects can be identified rapidly from their projections or even from their degraded projections. He also provides theoretical arguments for his selected set of geon components; this set is based on object representation in terms of 'nonaccidental' or '**viewpoint-independent**' properties. These are properties of the projections of objects which hold for the 3-D objects themselves. For instance, the straightness or collinearity of a line is a nonaccidental property, as it is usually the projection of a straight edge. The specific length of a line is not a nonaccidental property, as a short line does not necessarily refer to a short edge. It may be the projection of a long edge in a certain orientation in depth. In general, quantitative aspects are not considered as nonaccidental properties.

We will now show how the thirty-six geons can be deduced from properties that are assumed to be nonaccidental. There are properties that apply to the cross section of a geon and others that apply to the axis of a geon. The properties are illustrated in figure 14 by patterns that are variations of a regular prolonged cube.

Two types of edges are distinguished; a straight (S) one (shape A_1) and a curved (C) one (shape A_2). Three kinds of symmetry are distinguished. For one kind (+ +), a cross section is invariant under a 90° and 180° rotation (Garner 1974) as well as

under horizontal and vertical reflections (see shape B_1). For a second kind of symmetry (+) the cross section is *only* invariant under reflection (shape B_2). For a third (-), there is no *symmetry* in the cross section (shape B_3). Also, a constant (+) cross section (shape C_1) is distinguished from an expanding (-) cross section (shape C_2) and from one that is both expanding and contracting (-) (shape C_3). Finally, a straight axis (shape D_1) is distinguished from a curved axis (shape D_2). All combinations of these variations lead to the thirty-six geons which are shown in figure 15. In this figure, the organization of the geons is slightly different from that in figure 14.

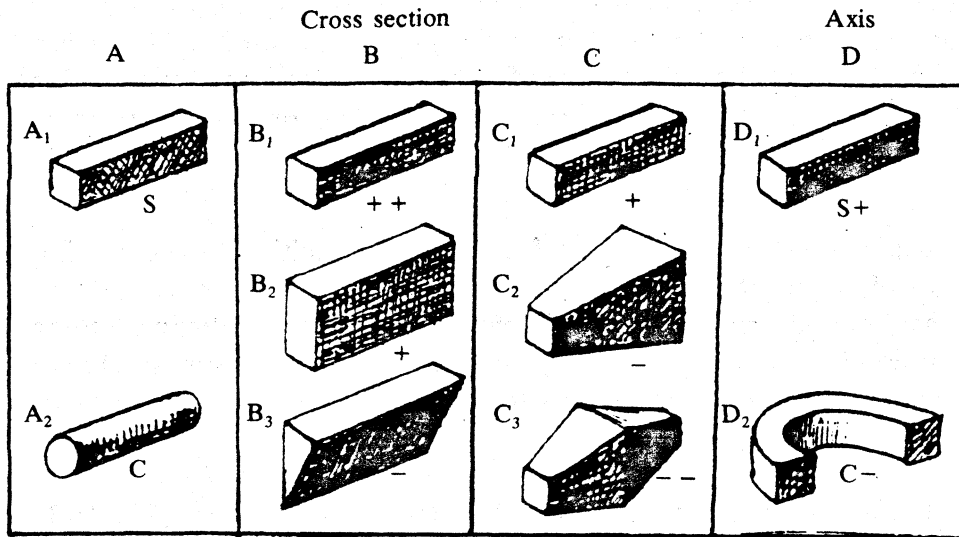


Figure 14. Partial set of geons, distinguished by various nonaccidental properties.

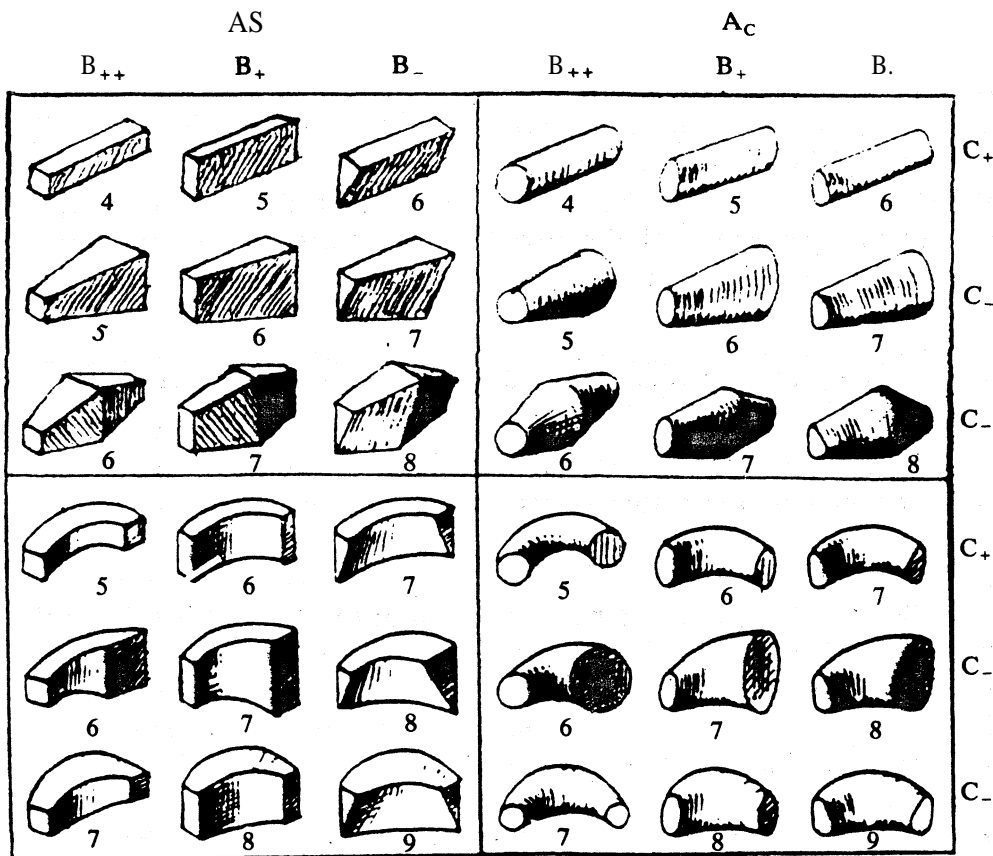


Figure 15. The complete set of thirty-six geons. The columns A and B refer to cross-section properties, and the rows C and D refer to side-view properties. The numbers refer to structural information loads. These loads are consistent with the + and - evaluations of Biederman.

Columns A, B, C, and D in figure 14 correspond to nested columns and rows in figure 15. The columns reveal the cross-section views, and the rows reveal the side views of the geons. In Biederman's taxonomy of geons, the axis is determined by its length with respect to the diameter of the cross section. Furthermore, the axis is preferably one for which the cross section is both constant and symmetrical. Finally, geons are separated by sharp angles, especially by concave angles. Herewith, we conclude a rough sketch of Biederman's model.

Notwithstanding the appealing aspects of the **RBC model**, we have some points of criticism. They mainly refer to the way this model explains object identification at the cost of object differentiation and organization. First, we focus on a few technical details of the model. Some of them have already been discussed by Biederman. Later on, more general and fundamental issues of the RBC model are considered and are contrasted with the DM model.

3.4 *Unclassifiable objects*

There are shapes that are not readily classifiable by geon coding. Figure 16a shows an object that expands and contracts simultaneously with respect to different dimensions. Hence, it cannot be classified as a single geon. It can indeed be described as a composition of an invisible geon, shown in figure 16b, and a negative invisible geon, shown in figure 16c. However, we question whether this subtraction can be obtained by a **bottom-up** identification of the two virtual geons. Another example of a shape that is difficult to classify is the spiral, shown in figure 16d. Its axis is curved in, say, a horizontal plane, but is straight in a vertical dimension. The shape shown in figure 16e can be seen as two expanding geons, an upper and a lower one. However, this figure lacks a sharp concavity, which suggests that it should be considered as a single but unclassifiable geon.

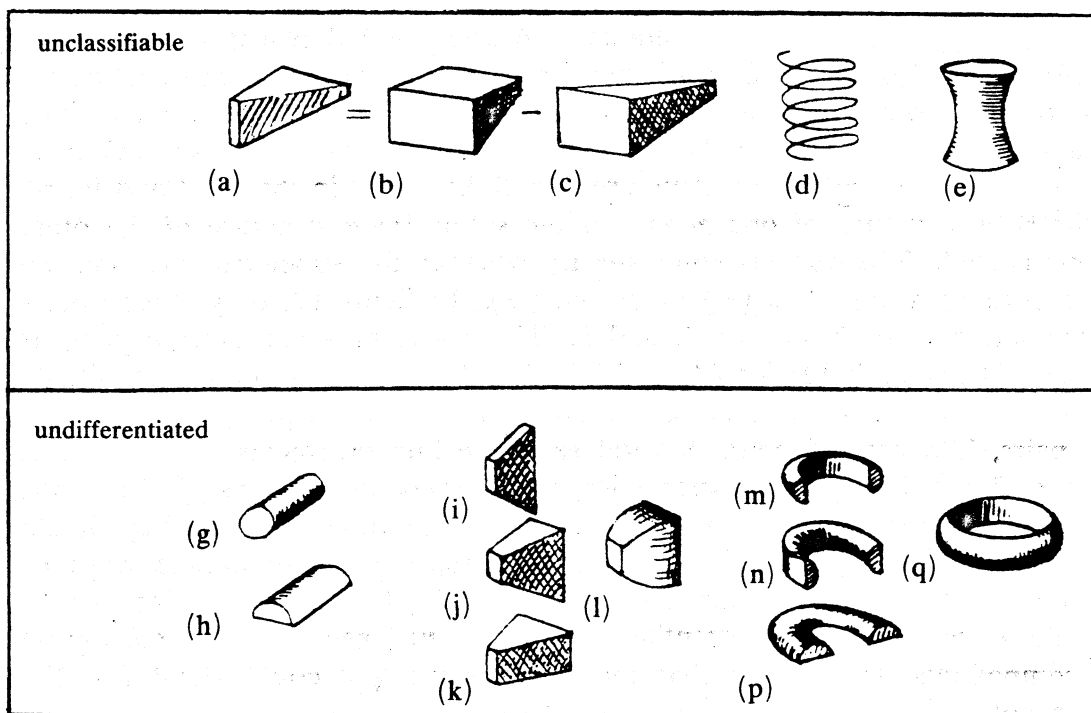


Figure 16. The shapes in (a), (d), and (e) are hardly classifiable in terms of geons. The shape in (a) can be conceived as a composition of an invisible geon shown in (b) and a negative invisible geon shown in (c). The shapes in (g) and (h) are classified as belonging to the same geon category. The same pertains for the set of shapes in (i), (j), (k), and (l), and for the set of shapes in (m), (n), (p), and (q).

3.5 *Undifferentiategeons and geon twins*

There are several shapes that are clearly different, **but** are classified as being the **same** according to geon **coding**. The cross section of the shape shown in figure 16h is both straight and curved. According to Biederman, this figure is a variant of that shown in figure 16g with a complete curved cross section. We do not object to this view, but prefer a more differentiated geon classification. Notice that many of the geons shown in figure 15 look even less different from each other than do the shapes in figures 16g and 16h. The same is true for the shapes in figures 16i, 16j, 16k, and 16l which are all classified into the **same** geon category. However, the way in which their cross sections expand does matter. Whether this expansion is straight or curved makes a lot of difference to the perceiver. A single glance suffices to reproduce each of these shapes quite well. In our view, the ability to reproduce is not independent of the ability to recognize shape. The shapes in figures 16m, 16n, 16p, and 16q belong to the same geon category too, yet they show varying relations between 'ventral' and 'dorsal' parts with respect to their global bodies. The closed shape shown in figure 16q is in fact perceived as distinct from an unclosed one, as with the characters C and O, and the numbers 3 and 8. Notice that, for all shapes, minimum codes can be made, from which these shapes can be reconstructed uniquely. The simplest codes of the shapes in figure 16 are all different (see later). The number of geons required to account for the differences between the shapes in figure 16 will, however, be **approximately** six hundred, instead of thirty-six.

We will now consider how pairs of geons that are classified as being the same appear completely different perceptually. In figure 17, shapes A, B, and C are geons that are classified as being the same and are variants of the shapes shown in figures 16m, 16n, and 16p. The geon axis of each of the shapes A, B, and C is curved in the horizontal plane. This corresponds to the **superstructure** of the simplest code and is indicated by **A1**, **B1**, and **C1** in figure 17. The cross section of each figure is the vertical curve. This curve is constant and corresponds to the subordinate structure, indicated by **A2**, **B2**, and **C2**. Combinations of A and B are shown in the first three rows of figure 17, and of A and C in the bottom row. The varying concatenations of two geons are selected such that a minimum of concavities occur. They are classified by the columns: in the first column the superstructures of the two geons are linked; in the second column the subordinate structures are linked; and in the third and fourth columns the superstructure of one geon and the subordinate structure of the other geon are connected. The various rows specify whether the superstructure and the subordinate structures are open (–) or closed (+). In figure 17, only three **open**-closed combinations are shown for A and B. There are, however, sixteen possible combinations. The same holds for AC and BC. In addition, A can be linked with A, B with B, and C with C. In fact, there are 288 pairs of such equal geons. Among these 288 pairs, 276 are different. Several resemble known objects such as vases, spoons, pipes, fishes, headphones, spectacles, mushrooms, and snakes. Most of the pairs are strange shapes. However, they can easily be identified. Hence, they should be represented in unique ways. They are indeed represented in unique ways by minimum codes. These codes are visualized in figure 17. However, according to **geon** coding, all these shapes are **undifferentiated** pairs of equal geons. In the analogy of phoneme components, this means that the single word '**pp**' could stand for 276 different concepts.

3.6 *Confusion with respect to unity and variety*

So far, we have discussed some examples in which geon coding is unable to distinguish different shapes. Now we focus on redundant aspects of geon coding which also

rise to confusion, especially with respect to unity and variety of shapes. First, we consider the RBC definitions of the axis and cross section:

- (i) The axis is longer than the width of the cross section.
- (ii) The cross section is preferably **constant**.
- (iii) The cross section is preferably symmetrical.

These three properties do not necessarily occur together. Ambiguity in the choice of axis or of cross section can be avoided by defining the axis by only one of these properties, eg its cross section should be constant. However, in that case, a whole cross-section category indicated in column C of figure 14 can be deleted. We will clarify this by means of an example; according to the RBC model, a cone has a straight axis and a circular expanding cross section, **but** according to the new definition of axis and cross section, the cone can be described as a triangular cross section with a circular axis. In fact, we described the expanding shape shown in figure 4c in this way. The redefined cross section then agrees with the subordinate structure and the circular axis with the superstructure.

The RBC definitions of the axis and cross section not only give rise to redundant coding, but also to confusion about unity and variety of shapes. We will attempt to demonstrate this. It was shown by van Bakel (1989) (as mentioned in section 2.8) that

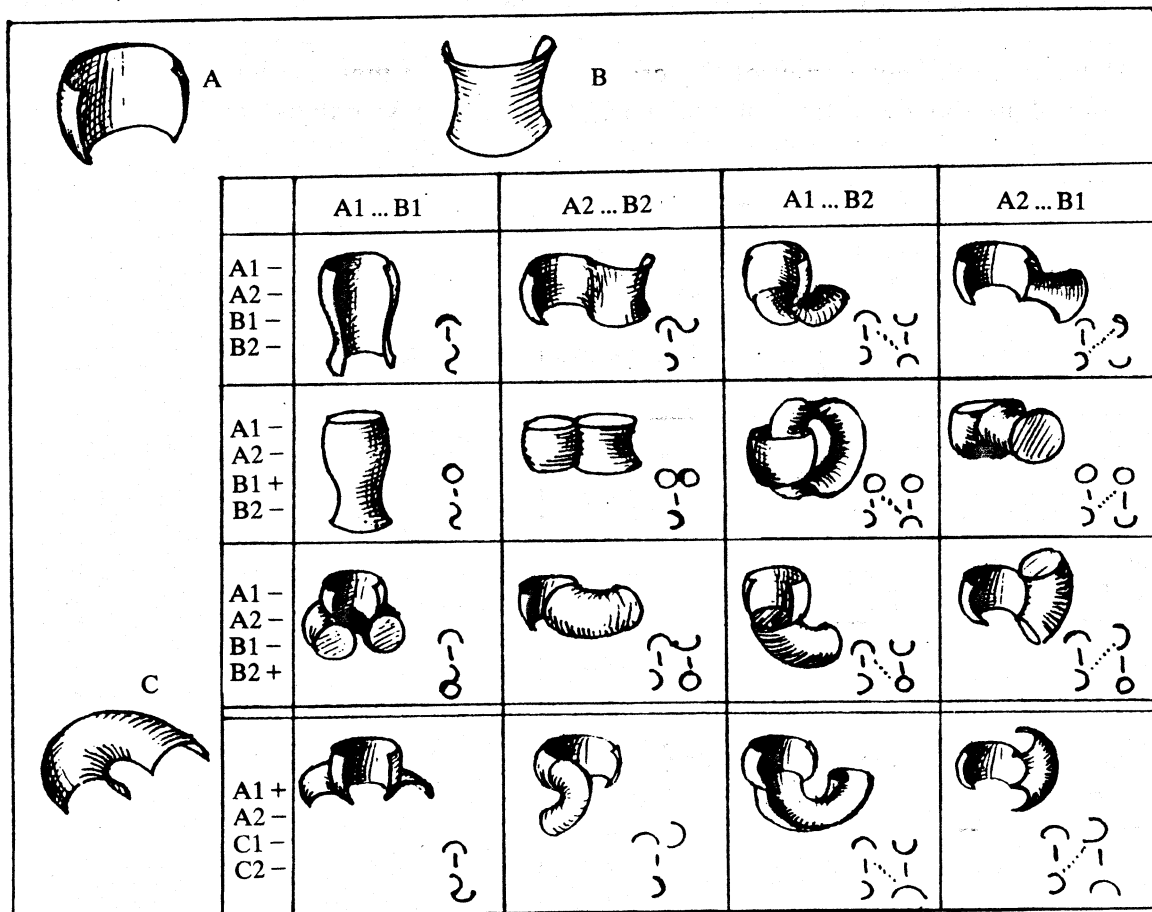


Figure 17. The sixteen shapes in the cells are pairs of equal geons. Only a subset of the 276 geon twins that look different are presented. These 276 can be generated by the various factors indicated in the figure. A, B, and C are equal geons. The three upper rows comprise some combinations of A and B; the bottom row comprises some combinations of A and C. The four columns refer to various concatenations of geons with a minimum of acute angles. The number 1 refers to the geon axis (superstructure) and number 2 to the cross section (subordinate structure). For instance, A1 ... B2 means that the axis of A is connected with the cross section of B. In each row, the + signs indicate closure. For instance B2+ means that the cross section of B is closed, ie circular. The geon pairs should look similar to each other, but they do not. Notice that the visualized minimum codes are indeed all **different**.

the shapes A, B, C, and D in figure 18 are all conceived as unitary shapes, in contrast to their counterparts A', B', C', and D'. Each of the eight shapes consists of two connected parts. Care is taken that the patterns within a pair, such as A and A', are roughly equal with respect to cusps and concave angles. Now the question is how might the RBC model account for the unity and duality of these shapes. Shape A is symmetrical about one common straight axis. This is not true for A'. This symmetry may be considered to be decisive for unity and duality and may also seem to account for the apparent unity of B and the duality of B'. In fact, their upper parts merely differ in a metrical-quantitative respect from the upper parts of the corresponding shapes A and A'. However, the axis of the upper part of shape B should be considered to be curved in the horizontal plane according to the RBC definition and to the new definition of the axis proposed above. But then the unity of B is not due to the symmetry around a common straight axis. Shape C will help us find the reason for the unity of shape B. The lower part of C also has a horizontally curved axis according to all definitions of the axis. Hence, the unity of shape C is not due to its symmetry around a vertical axis but to the common horizontally curved axes of both the upper and the lower part. The same holds for shape D and should hold also for B and A, if the new definition of axis is accepted. Notice that shape A can be extended in the horizontal plane, like shape D. In this extended form the unity of shape A cannot be attributed to a common symmetry around a straight axis. This implies that all upper and lower parts of the shapes A, B, C, and D have horizontally curved axes, merely defined by the constancy of the cross section. The metrical length should not be considered in the definition of the axis. In fact, the visualized codes shown in

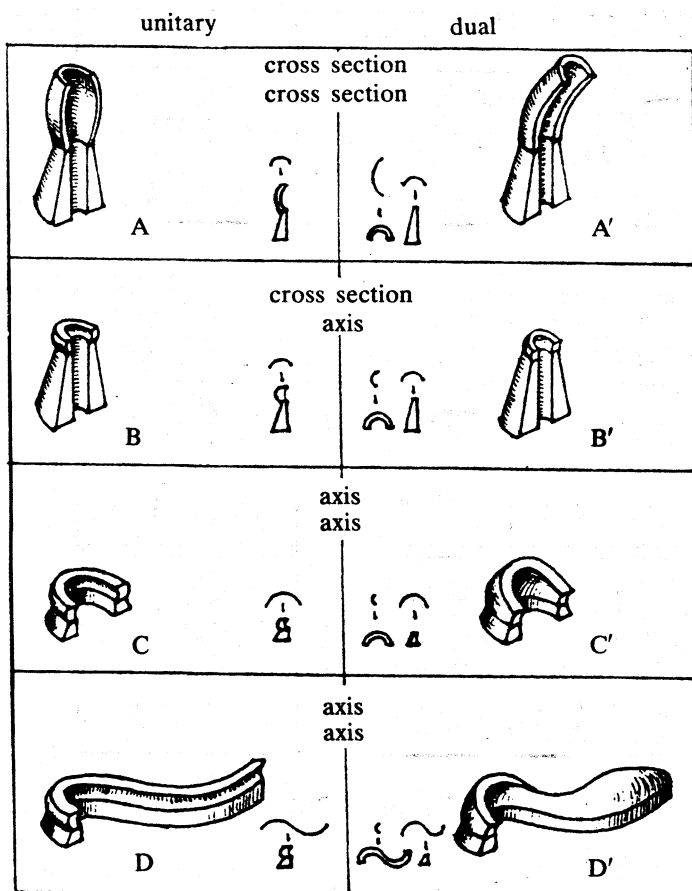


Figure 18, Shapes A, B, C, and D are conceived as unitary patterns and A', B', C', and D' as dual patterns. The superstructures of the codes, depicted to the left and to the right of the centre, reflect this **unity** and duality. If the superstructures are singular and undivided, unity is predicted, otherwise duality is predicted.

figure 18, and also in figure 17, reveal their unity or variety. If they **show** various and divided **superstructures** they **predict** variety, **otherwise** unity.

3.7 *Nonaccidental properties include biases towards simplicity*

Nonaccidental, or viewpoint-independent, properties can be considered as **projective-transformational** invariances in an a priori objective sense. However, from this perspective a proximal ellipse would refer to a distal ellipse instead of to a circle, or complex angularity rather than collinearity would be a viewpoint-independent property. An argument for the latter statement is that a proximal straight line can be a **projection** of a hook shape, but a proximal hook cannot be the projection of a straight line. This argument indeed holds under the assumption that retinal resolution is limited. Thus the perceptual eidotropic tendency (bias towards regularity) is not implied by an a priori objective meaning of viewpoint independency. This is because the invariance criterion for **figural** goodness, **proposed** by Garner (1974), does not pertain well to projective transformations.

In fact, Biederman does not consider viewpoint-independent properties in an a priori objective sense, but rather in an a posteriori subjective sense. **Semisymmetrical** proximal ellipses are supposed to refer to complete symmetrical circles, because ellipses are usually perceived as circles. This bias might rely on a general tendency towards simplicity. But if this is not accepted, this bias may stem from the fact that there are more circles than ellipses. In our view, this is true, at least for the man-made world in which we happen to live. However, in that case a completely different **RBC** model should be developed for a world with more ellipses than circles. It implies that regularity and irregularity are completely symmetrical, ie the first is a distortion of the second in the same way as the second is of the first. To us, this seems implausible.

Only one possibility remains; RBC is based on the minimum principle. A few indications are the following. Geons cover the simplest shapes almost hermetically; this appears from the structural information loads of the minimum codes of the geons, specified **according** to the syntax of Leeuwenberg (1971a, 1971b). These loads are indicated in figure 15. Although the minimum principle does not aim at a decomposition of a complex object into simple ones, it is obvious that a decomposition into complex elements runs the risk of creating a complex representation of the total object instead of a simple representation. These considerations lead us to believe that geons are a posteriori components of minimum codes, rather than a priori elements of codes. Some support for this point is given by the evaluation of the nonaccidental properties, given by Biederman. Some are positive, some negative, as shown in figure 14. Figure 15 shows that there is a remarkable correlation between this positive evaluation and the simplicity of the geons.

A short note on perspective should be made. Biederman disregards small deviations from **parallelity**: almost parallel lines are considered to refer to parallel lines. However, deviations due to perspective do in fact matter, depending on context (Perkins 1983; Cutting 1986). For instance, the central part of figure 19a is readily interpreted as a rectangle, but in figure 19b as a trapezoid.

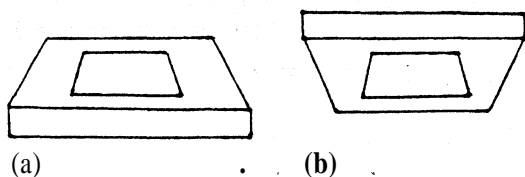


Figure 19. The central part in (a) is readily conceived as a rectangle. Although the central part in (b) is equal to that in (a) as a 2-D pattern, in (b) it is conceived as a trapezoid. Thus pattern interpretations are codetermined by the perspective of the context (Perkins 1983).

3.8 *Assumptions versus application domain*

The RBC model distinguishes about ten nonaccidental properties, which give rise to thirty-six **geons**. The geon decomposition of stimuli permits complex familiar objects to be rapidly distinguished, but not all **natural** or unfamiliar shapes. The DM model can describe all shapes uniquely. Nevertheless, DM distinguishes only two kinds of primitives, namely lengths and angles (in serial and bifurcated relationships). These primitives are common to all shapes. A series of primitives standing for a pattern reveals which primitives are identical and which are not. On a second level of pattern analysis, how identity relationships in a pattern can best be covered by 'basic' regularities is tested. Only three kinds of regularities are used (see section 2). Therefore, to us the DM model seems more parsimonious and atomic than the RBC model. The DM model permits description of all shapes, but also explains to some extent, various visual phenomena, such as unity and variety, foreground and background (Leeuwenberg and Buffart 1984), pattern completion (Buffart et al 1981), transparency (Leeuwenberg 1978), neon illusions (van Tuyl and Leeuwenberg 1979), assimilation and contrast (Leeuwenberg 1982), subjective contours (van Tuyl and Leeuwenberg 1982), embeddedness (van Tuyl 1980), and judged complexity (Leeuwenberg 1969; Simon 1972). The recognition of an object from its degraded projection is not exclusively based on geon decomposition. The completion of degraded but unknown patterns can also be tackled from the DM approach (Leeuwenberg 1978).

The DM model applies not only to 3-D and 2-D shapes but also to serial visual patterns (Restle 1970, 1979; Simon 1972). It is even, to some extent, relevant for melodic patterns (Leeuwenberg 1978; Collard et al 1981; Deutsch and Feroe 1981). At least, regularities play a role in the perceptual organization of melodic patterns (Jones 1976; Jones et al 1982; Dowling and Harwood 1985). It is implausible that these musical regularities are based on viewpoint-independent properties. The great value of RBC is the fact that it provides a process model for the specification of an object on the basis of its retinal image. Until now, within the DM approach, a process model has been developed only for series of symbols and 2-D contour patterns (van der Helm and Leeuwenberg 1986).

3.9 *Conclusions*

Perception models, prescribing stages from global to local (Navon 1977; Beck 1982) or from part to whole (**Biederman** 1987), presumably make use of nonaccidental properties. These models explain, to some extent, the rapid identification of objects from their retinal images. However, some of these models merely apply to restricted sets of stimuli. Furthermore, their account of perceptual unity and variety is rather insufficient, whereas these two qualities can be derived from the minimum principle without specific assumptions for various cases. Phenomena that are attributed to perceptual-precedence effects by stage models are assigned to postperceptual-**dominance** effects according to the minimum principle. For instance, geons can be considered as organizational parts of simplest object codes. These parts are rather more derivatives of codes than principles of coding.

4 **Summary and conclusions**

The minimum principle model and process models of perception are compared. In both approaches predictions are made about the perceived unity and variety of patterns. From the perspective of the minimum principle, the unity and variety of patterns is mainly determined by the superstructure components of the simplest pattern codes. The superstructure is supposed to be dominant with respect to the subordinate structure, as the latter depends on the former code component and not

the reverse. The superstructure does not necessarily describe the largest global-pattern shape, neither does the assessment of the superstructure necessarily precede the assessment of the subordinate structure in the process from pattern towards its representation. In the reconstruction of the pattern from its code this order is indeed efficient. In addition to structural aspects (determined by identical versus non-identical pattern elements), metrical aspects (ie quantitative **aspects** such as size) may play a role in the selection of the simplest pattern code and hence in the determination of the dominant superstructure. However, this merely applies in case a pattern lacks a definite structural hierarchy.

Three process models are discussed. In these models predictions are made about the perceived unity and variety of patterns, especially in the case when they are considered as linear stage models. One **model** refers to Navon's global precedence hypothesis. The preferred organizations of disjunct texture-alike patterns can be explained by this hypothesis, but also by the minimum principle on the basis of the dominant superstructure assumption. Namely, for a 2-D texture-alike pattern the global shape coincides accidentally with the superstructure. A second model refer to Beck's prefocal orientation-detection hypothesis, which explains the preferred organization of unsegmented texture-alike patterns. These organizations can also be explained, at least partly, by the minimum principle on the basis of the dominant superstructure hypothesis, but in a less direct way; the actual superstructure of a conjunct texture-alike pattern is not decisive about the pattern organization, but the one but highest hierarchical level is decisive. This level describes the orientations of subpatterns. A general objection to the two models mentioned **above** is that they apply to specific kinds of patterns. As a consequence, each of the models presupposes a process stage which tests whether a given pattern belongs to the domain of the model. This stage ought to precede the first stage prescribed by each of the stage models.

A third model refers to Biederman's recognition-by-components theory. This model makes use of nonaccidental or viewpoint-independent pattern properties for the rapid identification of geons in complex objects. First, we have attempted to show that there are several simple objects which are not well classified and differentiated by geons. The geon coding causes an explosion of undifferentiated geon twins. Moreover, the perceived unity and variety of geon pairs do not appear from their geon descriptions. The goal of **RBC** seems different from that of the descriptive minimum principle; RBC aims at a rapid distinction between complex objects, whereas the DM approach focuses on a full account of the internal structure of each shape. However, these two different goals do not give rise to the assumption that objects are coded by two kinds of representation. We doubt whether viewpoint-independent properties really refer to objective invariances. They probably refer to perceptual biases towards simple and regular structures. At least the positive values given by Biederman to some nonaccidental properties relate surprisingly well to simplicity. The DM model accounts, with fewer assumptions, for more perceptual phenomena and illusions than does RBC. Furthermore, the basic regularities are relevant in more different domains of cognition than are viewpoint-independent properties, eg in music. In contrast, RBC accounts for the process of identification of objects from retinal images. Up till now the minimum principle did not provide such a process model.

Our final conclusion is that the selection of a pattern representation is governed more by the descriptive minimum principle than by the three process models discussed here.

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