

THE IMPACT OF GESTALT PRINCIPLES ON SOLVING GEOMETRIC PROPORTIONAL ANALOGIES

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ABSTRACT

Analogy making is a highly sophisticated task and requires intelligence, since analogous patterns in two given stimuli are often not obvious and require a certain conceptualization. In this paper, we investigate solution strategies for ambiguous proportional geometric analogies. We constructed different analogies which were varied in such a way that they trigger different perceptions. Different solutions were constructed when subjects perceived different dominant structural patterns. The experiment investigates preferred solutions for these analogy variations, analyzes the influence of Gestalt principles on the perception of structural patterns in geometric figures, and examines how this influence leads to different solutions for the same analogy problem.

INTRODUCTION

Analogy making is a high-level cognitive process using knowledge from a source domain to solve a new problem in a different situation (the target domain). A common analogy pattern is the proportional analogy of the form $(A:B)::(C:D)$, where A, B and C are given elements and the analogy is completed by inserting a suitable element for D. To solve this kind of analogy one has to analyze the relation between the source elements A and B and then apply it to the target element C to construct D. Figure 3 shows proportional geometric analogies (GPA) as they are commonly used in intelligence tests. Often different geometric figures exist which are all possible solu-

tions, but might not be equally plausible and depend on the perception of the elements A, B, and C (Schwering et al., 2007).

When the human visual sensory system observes a geometric figure, it transforms the unstructured information into a structured representation of coherent shapes and patterns. Human perception tends to follow a set of Gestalt principles for organization: stimuli are experienced as a possibly good Gestalt, i.e. as regular, simplistic, ordered and symmetrical as possible (Koffka, 1935; Köhler, 1929; Wertheimer, 1912). The Gestalt psychology identified different principles according to which humans construct Gestalts (e.g. principle of proximity, similarity, symmetry and good continuation). Applying different Gestalt principles to the same geometric figure might result in different perceptions: Although there have been identified certain rules which Gestalt principles are cognitively preferred, there does not exist a fixed hierarchy of Gestalt principles. The perception of geometric figures may differ among humans and among contexts.

The human subject test described in this paper investigates preferred solutions for ambiguous GPAs with multiple plausible solutions. The GPAs are varied such that different perceptive interpretations might be triggered. We hypothesize that people perceive different structural patterns in the same series of geometric figures and different structural patterns lead to different solutions. We analyze the solutions created by the subjects to find a plausible interpretation for the different perceptions by different dominant Gestalt principles. These results are important to understand the

human strategies of solving analogies in a better way, and furthermore to develop heuristics for automatic analogy models to solve geometric analogies in a cognitively plausible way.

The remainder of the paper is structured as follows: the second section sketches related experimental work on proportional analogies. Section three explains the method of the experiment, focusing in particular on the stimuli used. Then the results of the experiment are presented and discussed in the following sections. Finally, we conclude and give different directions of future work.

RELATED EXPERIMENTS

Proportional analogies have been investigated already by Aristotle in historic times. In modern psychology, particularly since (Spearman, 1923), GPAs are applied to measure human intelligence. In Raven's Progressive Matrices Test (1958), participants are supposed to complete 3x3 or 2x2 matrices of geometric figures¹. The completion of these matrices requires analogy-like analysis processes: determination of visiospatial relations between figures, establishment of analogical mappings and transfer of relations (Carpenter et al., 1990). GPAs were also tested in various computational approaches: Evans (1962) developed a system to compute GPAs from the 1942 and 1943 American Council on Education Examination (ACE) test, Tomai et al. (2004) tested the analogy model SME with geometric Millers Test Analogies, and Lovett et al. (2007) with Raven's Progressive Matrices. Mulholland et al. (1980) investigated analogies consisting of complex line drawings. They varied the information structure (number of elements in a geometric figure and number of required transformations) in the analogies systematically.

The geometric analogies have a proportional structure as in our experiment, but the stimuli differ: While our experiment contains

¹ The problems in Raven's Progressive Matrices have a different structure, but apply mechanisms of proportional analogies.

only not-overlapping, colored triangles, circles, and squares, the geometric figures in the other experiments typically consist of lines, points, or areas. Some of them are simple (e.g. a single line) and some of them are complex. ACE analogies have similar shapes like our stimuli, but elements can be inside other elements. In our experiment, subjects constructed their solution. This is different from the set-up in standard intelligence tests, where subjects select from pre-defined solutions. Letting the subjects construct their own solutions does not constrain their reasoning processes to a limited set of answers, which is particularly important for ambiguous analogies. Moreover, having the freedom of constructing solutions causes the subjects to reflect more cautiously on which transformations are necessary for them to complete the analogy, in contrast to selecting from pre-defined solutions, which might be accomplished by only considering superficial features and not the relational structure. Another major difference between previous tests and our investigations are the systematic variations and the ambiguity of the our GPAs: The geometric figures of the source or the target are varied to trigger different Gestalt perceptions. All analogies are ambiguous and allow for different plausible solutions. Different solutions occur when different structural patterns are perceived in the figures.

METHOD

Participants

The participants were visitors of two public events, the Technologietag 2007 (TT) and the Hochschulinformationstag² (HIT), who volunteered to take part in the experiment. In total there were 112 participants, from which 103 usable datasets were collected (50 from the TT and 53 from the HIT). The participants'

² The Technologietag 2007 was an exhibition for people interested in computer science and technology and the Hochschulinformationstag is an annual exposition day of the study programs at the University of Osnabrueck intended for pupils.

age ranged across all age-groups – from “younger than 20” to “over 40” years. 50 participants were female, 52 were male, and one did not specify gender. 74 participants (72% of total number) were Germans; the others were from countries worldwide. The participants were not paid for their contribution to the experiment.

Material and Design

We created 21 analogies built out of circles, squares, and triangles in three different colors. Three out of the 21 analogies were “original analogies” and eighteen variations. Each original analogy was varied three times in the source, and three times in the target domain.

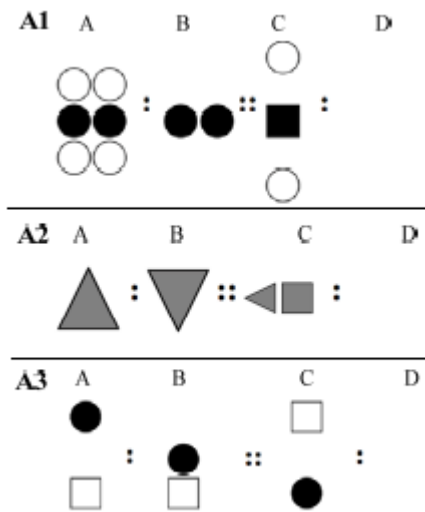


Figure 1: The three original analogies.

Figure 1 shows the “original analogies” and Figure 2 shows two variations of analogy A3 (c.f. appendix for complete overview). All analogies were ambiguous and varied to emphasize Gestalt laws to a different degree and trigger different solutions. We focused on four Gestalt principles: proximity, similarity, symmetry and good continuation. The extent of

varying the analogies was limited to position, shape, number of objects, and color.

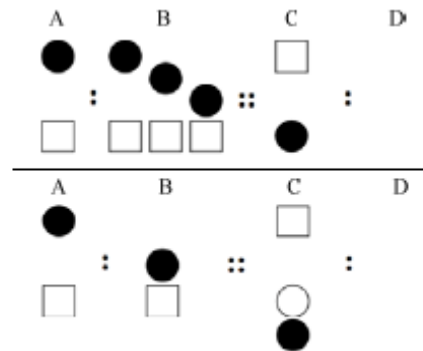


Figure 2: Variations of the original analogy A3 in the source domain (top) and in the target domain (bottom).

Figure 2 (top) shows an example of a variation in the source domain of the original analogy A3. Adding new elements to part B of the source domain (A:B) creates a new notion: guided by the Gestalt law of good continuation humans might perceive the circles to be arranged in a descending order. The second variation in Figure 2 (bottom) adds a new element to the target domain, causing a change in the way of perceiving, which is this time triggered by the principle of proximity: the new element (white circle) creates a new relation by means of proximity.

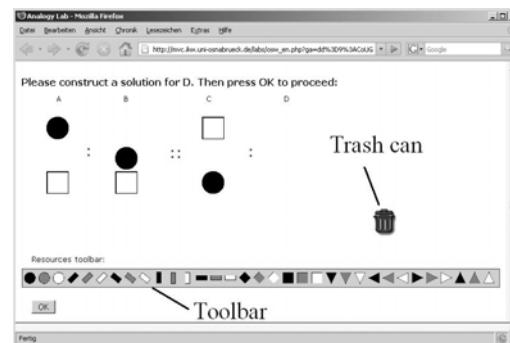


Figure 3: Screenshot of the Analogy Lab while creating a solution, with additionally labeled toolbar and trash can.

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In the experiment we used a software tool especially developed for this purpose: the Analogy Lab³ (Figure 3) is a web-based framework for constructing solutions to geometric proportional analogies via drag&drop. It also provides a clustering algorithm to classify the solutions constructed by the participants.

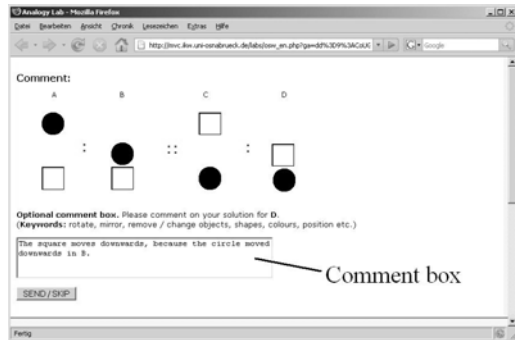


Figure 4: Screenshot of the Analogy Lab while writing in the optional comment box.

For the construction of the solutions a toolbar with all necessary geometric objects was available and there was a trash can symbol for throwing away undesired objects. The objects in A, B and C were not movable. There was neither a time limit to construct the solutions, nor was there a limit to the number of objects that could be used. The Analogy Lab was running on laptop computers (2 at the TT and 3 at the HIT) with a 15" screen and a resolution of 1024x768 pixels or above.

Procedure

The subjects' participation in the experiment began as they sit down in front of one of the laptops. The Analogy Lab browser window showed the starting page of the experiment. The experiment took place in a semi-controlled environment: an experimenter was present at all times, but people passed by in the vicinity of the experiment site.

³ <http://mvc.ikw.uos.de/labs/cc.php>

The experiment consisted of solving a sequence of analogy problems, and each solution consisted of two phases: the construction phase and the comment phase. At the beginning, the participants went through a one-example practice session. In the construction phase (Figure 3) subjects saw the first three (A, B, [source domain] and C [target domain]) objects of an analogy and solved the problem by constructing the object D in order to complete the analogy. The input was made by drag&drop with a standard mouse. The object could be dragged to the trash can symbol if not desired. Once the subjects were satisfied with their solution, they clicked on the OK button to proceed to the second phase (Figure 4), where subjects were asked for optional commenting on their constructed solutions. The subjects were able to see what they had constructed while they were commenting. They were supposed to type a short description of the steps of reasoning they had made in the process of constructing the solution. In order to facilitate and shorten the process of commenting, some keywords were suggested above the comment box, such as: rotate, mirror, remove/change objects, shapes, colors, position etc. Again a button click was required, this time for proceeding to the next analogy.

The stimuli were presented in a pseudo-randomized manner to avoid order effects. The set of stimuli was divided into three combos: each combo contained one out of the three original analogies, and two variations of each of the three original analogies. One combo was randomly selected for each participant. Thus each participant had to solve seven analogies in total. As soon as a participant had finished the test, a message on the screen indicated test termination.

RESULTS

For each participant the constructed solutions and the comments were stored in a log file. The log file was then parsed and the data was clustered to group the same solutions and compute the preference degree. The comments were evaluated with the aim of identifying

structural patterns and transformations used by the participants in the process of creating the solutions, and furthermore to determine the strategies for solving geometric analogies.

There were 31-37 participants per combo. The answers for one original analogy and for its respective variations of the source and target domain were compared. We will focus here on analogy “A1Original” (Figure 1) as well as two of its variations. The appendix comprises the complete results for the remaining analogies.

The results for the original analogy “A1Original” are depicted in Figure 5: 93% of the participants constructed the solution shown with the letter (a) while the rest created other solutions, labeled with (b) and (c) in the figure. 87% of the participants who constructed solution (a) provided a comment describing their opinion on how this solution was created. The analysis of the comments revealed that the participants applied different transformations to construct the same solution. The participants explained their solutions with the following transformations: 38% of the participants argued that they left the middle object intact, 27% stated that they removed the outer objects, another 27% removed the white circles, whereas 8% stated that the black square should stay. Obviously, all participants perceived the same structural pattern, namely a group of white circles and a group of black circles (a group of middle objects and a group of outer objects leading to the same pattern).

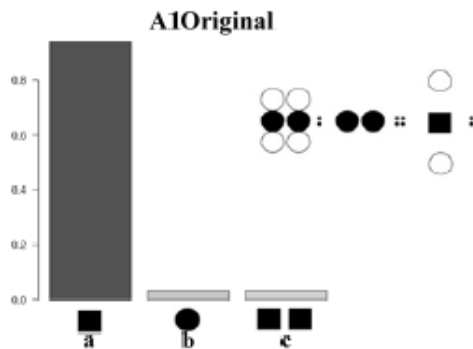


Figure 5: Original analogy “A1Original”

Figure 6 shows the results for analogy “A1S3”, one of the variations of the analogy “A1Original”. In this variation the source domain has been altered by moving the black circles in part A from the center to the top while the rest of the analogy was kept unchanged. Here 44% of the participants constructed the solution (a), 38% solution (b) and the rest created sundry solutions. 93% of the participants who constructed solution (a) provided comments. All of them argued that they removed the white objects (i.e. they perceived a group of black and a group of white objects). For solution (b) 84% of the participants provided a comment and described the following transformations: 55% kept the first row objects (i.e. they grouped the top and the remaining objects), 15% rotated in a 3D manner (i.e. they perceived the whole figure as one group), 15% moved the upper row one position down (grouping of top and remaining objects).

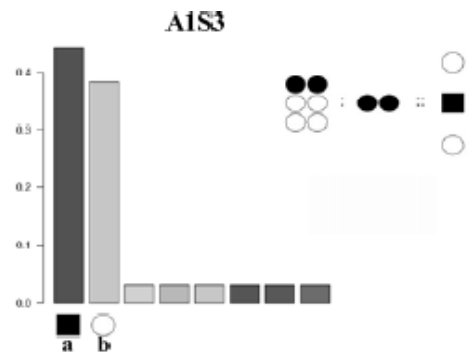


Figure 6: Results for a variation of the source domain

Figure 7 shows the results for analogy “A1T3” which is another variation of “A1Original”. Here, the target domain has been altered by inserting additional elements. Now 50% of the participants constructed solution (a), 35% solution (b), 10% solution (c) and the rest created sundry solutions. 76% of the participants who constructed solution (a) commented on it and from their descriptions the following transformations were identified: 85% kept the center object (grouping of center

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and outer objects) and 15% removed all types of circles (grouping based on shape). All participants who constructed solution (b) argued that they took the white figures away (i.e. grouping based on color). 66% of the creators of solution (c) provided comments and stated that the outermost circles had to be removed (again grouping of outer and inner objects).

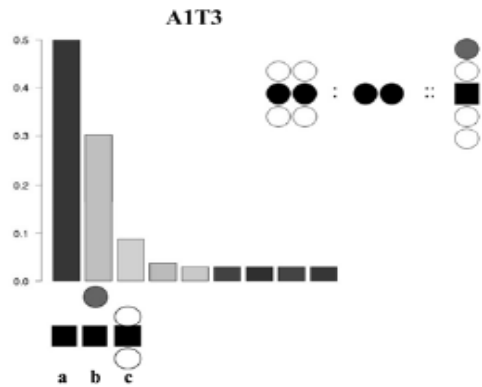


Figure 7: Results for a variation of the target domain

The results for the remaining variations of “A1Original” are summarized in Table 1. The analogy tasks are depicted together with the preferred solutions. For each solution the number of participants who created that solution, the transformations that were described in the comments and the perceived structural pattern (cf. discussion below for details on the analysis) are given.

Discussion

This experiment investigates whether humans see different structural patterns and relate them to preferred solutions. We wanted to investigate the solution strategy and explain the differences in terms of Gestalt psychology. The stimuli in our experiment were designed in such a way that perception of different structural patterns and application of different transformations would lead to different solutions. The results show clearly that the perception of figures differs among participants.

Table 1: Results for “A1Original” and Variations

Analogy A1S1			
Solutions	Comments	Pattern	
■	24/31 (77.4%) Center object stays Remove outer objects	III III	
○	4/31 (12.9%) Remove object	V	
Analogy A1S2			
Solutions	Comments	Pattern	
■	28/37 (75.6%) Center object stays Remove outer	III III	
○	3/37 (8.1%) Remove outermost objects Remove gray objects	III I	
●	2/37 (5.4%) Divide vertically by two	VI	
Analogy A1T1			
Solutions	Comments	Pattern	
■	12/31 (38%) Center object stays	III	
●	7/31 (22.5%) Remove all white objects	I	
○	4/31 (12.9%) Remove outermost objects	III	
●	3/31 (9.6%) Keep all black circles	V	
■	3/31 (9.6%) Remove all non black objects	I	
Analogy A1T2			
Solutions	Comments	Pattern	
○	28/37 (75.67%) Center object stays Remove outer object	III III	
■	3/37 (8.1%) Remove white objects	I	

From the comments and from the solutions we can infer that the subjects perceived different structural patterns leading to a different structural representation of the figures. Also, subjects applied different transformations. These findings agree with results from another study on re-representation: Similarly to this experiment, Kokinov et al. (2007) could show spontaneous re-representation during solving proportional analogies with ambiguous figures.

Structural Patterns and Gestalt Psychology

The comments showed that the subjects often applied transformations to several objects at a time, which indicates that they formed groups of objects and perceived structural patterns which can be explained in terms of different Gestalt principles. The results reveal a set of different strategies for building structural representations.

- I. Subjects used color as discriminating factor between groups, e.g. they constructed a group of black or white objects. This pattern can be explained by a dominance of the similarity Gestalt principle, caused by common color.
- II. Subjects used shape as discriminating factor between groups, e.g. group of circles, triangles, or squares. This pattern is explained by the similarity Gestalt principle.
- III. Subjects used relative position to form coherent groups, e.g. top objects versus remaining objects, center versus outer objects, or the left/first object and the right/second object⁴. This pattern is supported by the Gestalt principles of spatial proximity and of similarity.
- IV. Subjects used constant direction of movement of similar objects as the black

circles in analogy A3S1. This pattern can be explained by the Gestalt principle of good continuation.

- V. Subjects used a combination of several criteria to form groups. However, this occurred only in complex analogies.
- VI. Subjects perceived a whole figure as one single group and applied the transformation to the whole figure (no grouping).
- VII. Subjects perceived every object as separate.

In some cases the target domain contains an object with a new feature which did not occur in the source domain. For example, in analogy A2S2 the target domain contains grey elements. In this case, subjects adapted their grouping and added the object with the new feature to one of the existing groups (e.g. with a distinction between black and white objects, the grey objects were assigned to the black group which became a dark or colored group).

In very few cases, people mapped objects of the source on the empty set of objects in the target. For example, in analogy A1S2, some people commented on deleting the grey objects in the second solution. Since the target domain does not contain any grey objects, figure D is the same as figure C. In general, however, this “empty” mapping is avoided.

Transformations

The GPAs in this study could be solved by comparing figure A and figure C for common structures and establish a mapping between A and C. Then, the transformation from figure A to B is transferred and applied to figure C to construct figure D. The following transformations are mentioned by our subjects (we list only transformations if they were applied at least two times by different participants and rephrase them, if the same transformation is meant):

- rotation of a group of objects. Center of the rotation was the center of the group or a fixed point in the figure
- reflect a group of objects along a horizontal/vertical axis which is defined relative

⁴ We have to point out that all stimuli consist either of a repetition of (almost) the same figure or an iteration of several objects. For testing 2D spatial relations we need to extend the stimuli set with figures where objects change and move equally in the horizontal and vertical dimensions.

to the objects or absolutely in the coordinate system.

- keep or remove a group of objects
- move a group of objects towards a relative/absolute point (e.g. center of figure) and move objects in a continuous movement.

This set of transformations is similar to previous investigations from Evans (1962) and Dastani et al. (1998)⁵. We will use this set of structural patterns and applied transformations for the development of a computational model for GPAs (Schwering et al., 2009).

SUMMARY AND FUTURE WORK

This experiment investigates the Gestalt-perception of humans solving GPAs. We examined different plausible solutions for ambiguous analogies which are based on different structural patterns and different transformations perceived by the subjects. By varying parts of the analogies we found that preference degrees vary. This can be explained by the influence or dominance of different Gestalt principles. In some cases the grouping is influenced by the context of the analogy. For example, in analogy A1T3 solution (a) 15% grouped according to shape. Since only figure C comprises objects with a different shape, this grouping strategy must be triggered from figure C.

From the experimental data we could infer different strategies for solving the analogies, in particular different grouping strategies and different transformations. These results can be used to develop suitable heuristics such that a computational model can simulate the same results as found in our human subject test.

ACKNOWLEDGEMENTS

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⁵ This is different from the GPAs in Evan's approach, where transformations are used to relate figure A to figure C.

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APPENDIX

Table 2: Results for “A2Original” and Variations

Analogy A2Original 			
Solutions	Comments	Pattern	
 16/32 (50%)	Rotate 180° Flip image Vertical reflection	VI VI VI	
 9/32 (28%)	Mirror image Horizontal reflection	VI VI	
 6/32 (18%)	Rotate triangle 180°	II	
Analogy A2S1 			
Solutions	Comments	Pattern	
 20/32 (63%)	Rotate triangle 180° Rotate left object	II III	
 8/32 (25%)	3D-Reflect triangle horizontally	I	
 2/32 (6.5%)	Rotate grey objects 180° Triangle changes position	I II	
Analogy A2S2 			
Solutions	Comments	Pattern	
 31/32 (97%)	Vertical reflection Rotate 180° Mirroring	VI VI VI	
Analogy A2S3 			
Solutions	Comments	Pattern	
 28/36 (77.7%)	Reflect obj. separately Rotate each figure 180°	VII VII	
 7/36 (19.4%)	Rotate 180° Reflect	I	

Continuation of Table 2:


















Analogy A2T1 			
Solutions	Comments	Pattern	
 31/35 (88.5%)	Mirroring horizontally Rotate as whole 180° Invert	VI VI VI	
 3/35 (8.5%)	Mirror individual figures	VII	
Analogy A2T2 			
Solutions	Comments	Pattern	
 16/32 (50%)	Rotate as whole 180°	VI	
 7/32 (21.8%)	3D-Rotate as whole 180° horizontally	VI	
 3/32 (9.3%)	Rotate only triangles 180°	II	
Analogy A2T3 			
Solutions	Comments	Pattern	
 16/37 (43.2%)	Mirroring horizontally	VI	
 9/37 (24.3%)	Rotate as whole 180°	VI	
 7/37 (18.9%)	Rotate individually 180°	VII	

Table 3: Results for “A3Original” and Variations

Analogy A3Original 			
Solutions	Comments	Pattern	
 22/37 (59.4%)	Move square downward	II	
 15/37 (40.5%)	Move circle upward	II	
Analogy A3S1 			
Solutions	Comments	Pattern	
 20/37 (54%)	Upper object moves down Swap color and shape	III V/I/II	
 12/37 (32.4%)	Flip as whole object 180° horizontally	VI	

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Continuation of Table 3:

Analogy A3S2 		
Solutions	Comments	Pattern
 23/34 (67.6%)	Move square downward	II
 12/34 (35.3%)	Move circle upward	II
Analogy A3S3 		
Solutions	Comments	Pattern
 21/32 (65.6%)	Upper object moves down	III
 5/32 (15.6%)	3D-Flip as whole object 180° horizontally	VI
Analogy A3T1 		
Solutions	Comments	Pattern
 21/37 (56.7%)	Circles move upwards	II
 10/37 (27%)	Square moves downwards	II
 2/37 (5.4%)	Only Black circle moves upwards	V/I
Analogy A3T2 		
Solutions	Comments	Pattern
 23/34 (67.6%)	Square moves downwards	II
 10/34 (39.4%)	Black moves to white Circles move upwards	I II
Analogy A3T3 		
Solutions	Comments	Pattern
 24/32 (75%)	Upper figure moves down	III
 4/32 (12.5%)	Black moves to white	I

People commented on the transformations applied. Please note that these transformations do not necessarily lead to the solution they constructed.